Logarithms

Definition of the Logarithmic Function

$\log_a x = y \quad \Leftrightarrow \quad a^y = x$ where "a" is a positive number and $a \neq 1$

In words, $\log_a x$ is the exponent to which the base "a" must be raised to give "x".

Example: $\log_2 8 = 3 \quad \Leftrightarrow \quad 2^3 = 8$

Logarithmic form *Exponential form*

The word "log" asks: What power do I put on 2 to get 8? *Answer: 3*

Common Logarithm: The logarithm with base 10 is called the *common logarithm* and is denoted by omitting the base: $\log x = \log_{10} x$

Natural Logarithm: The logarithm with base *e* is called the *natural logarithm* and is denoted by **ln**:

$\ln x = \log_e x$ $\ln x = y \quad \Leftrightarrow \quad e^y = x$

Properties of Logarithms

Examples

1. $\log_a 1 = 0$ $\log_5 1 = 0$
2. $\log_a a = 1$ $\log_5 5 = 1$
3. $\log_a a^x = x$ $\log_5 5^8 = 8$
4. $a^{\log_a x} = x$ $5^{\log_5 12} = 12$

Laws of Logarithms

Examples

Let "a" be a positive number, with $a \neq 1$. Let A>0, B>0, and C be any real numbers.

1. $\log_a (AB) = \log_a A + \log_a B$ $\log_5 12 = \log_5 (3 \cdot 4) = \log_5 3 + \log_5 4$
2. $\log_a \left(\frac{A}{B}\right) = \log_a A - \log_a B$ $\log_2 5 = \log_2 \left(\frac{15}{3}\right) = \log_2 15 - \log_2 3$
3. $\log_a (A^c) = C \cdot \log_a A$ $\log_4 6^3 = 3 \cdot \log_4 6$

   (*bring the power down to coefficient position)*

Change of Base Formula

Example

This formula allows you to find the calculator value of the log of any base.

$$\log_b x = \frac{\log_a x}{\log_a b}$$

$$\log_9 20 = \frac{\ln 20}{\ln 9} = \frac{\log 20}{\log 9}$$

change to: *base e* *base 10*