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Working Paper 11 - 01

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Volatility, Persistence and Nonlinearity of Simulated DSGE Real Exchange Rates

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Abstract

This paper investigates the time series properties of real exchange rates series produced by DSGE models. We simulate a variety of new open economy DSGE models that incorporate features such as local currency pricing, home bias, non-traded goods and incomplete markets. We attempt to ascertain whether the dynamics of the real exchange rate in this class of models are consistent with those found in the time series literature using data from the current floating period. Although none of the basic specifications we consider match the volatility in the raw data, our findings suggest that home bias in consumption and non-traded goods are the key components of DSGE models that are able to generate persistent real exchange rates, comparable to that in the data. Moreover, we find that some of the structural micro-level nonlinearity embedded within DSGE models may be represented as macro-level nonlinearity in the form of a smooth transition autoregressive process, which has previously been found to parsimoniously characterize the dynamics of real exchange rates in the time series literature.

JEL Classification: F41 F47 C15 C32

Keywords: Simulation, real exchange rate dynamics, nonlinear dynamics, smooth transition estimation, DSGE modeling

Current Version: July 21st, 2011

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1 Introduction

Understanding the behavior of exchange rates has always been an issue of great importance for central bankers and policy makers, more so in today's world characterized by the real and financial linkages that exist between countries. For example, the large and growing international current account imbalances faced by many developing and developed nations are prompting researchers to look for the causes and effects of these phenomena. However, any type of investigation into the international linkages that exist between countries, be it international capital flows, optimal currency areas or cross-country trade patterns, all clearly hinge upon a proper understanding of the dynamics of the real exchange rate. Over the course of the last decade, dynamic stochastic general equilibrium (DSGE) models have become very popular as a tool amongst both researchers and policy makers, and have been used extensively to study the implications of these international linkages on consumer welfare and policy conduct. Yet, all the policy prescriptions derived using the DSGE framework are conditional on the models' ability to replicate the observed facts about real and nominal exchange rates.¹ The contribution of this paper pertains to an evaluation of the DSGE framework with regards to its ability to yield real exchange rate dynamics that are consistent with the data. We focus on the issues of volatility, persistence and nonlinear dynamics.

Since the seminal paper by Rogoff (1996), in which he asked '...How can one reconcile the enormous short-term volatility of real exchange rates with the extremely slow rate at which shocks appear to damp out?', researchers have sought to resolve the purchasing power parity (PPP) puzzle present in empirical work. The open economy literature has taken two distinct approaches to the problem. On the theoretical side, DSGE models were updated with a variety of monetary and real frictions to improve the dynamics of simulated exchange rates. Empirical work, on the other hand, has

¹A non-exhaustive list of papers in this area includes Benigno and Benigno (2003), Corsetti (2006) and Devereux and Sutherland (2008).

focused on developing better time-series and panel estimation techniques, in particular suggesting that real exchange rates may exhibit nonlinearities in their behavior.² Our paper aims to bridge these two distinct strands of literature by studying the time-series properties of real exchange rates generated by the several widely used variants of international DSGE models.

To date, the vast majority of the theoretical literature employing the DSGE framework has focused on studying second moments of the real exchange rate (its variance and autocorrelation, as well as correlations with key macroeconomic indicators).³ In evaluating this framework, Chari, Kehoe and McGrattan (2002), henceforth CKM, were one of the first to point out that DSGE models with sticky prices and deviations from the law of one price were unable to replicate the observed volatility and persistence of real exchange rates. Additionally, the authors noted that exchange rates and consumption were far less correlated in the data than in the existing models of exchange rate determination. Several solutions have since been put forth to increase the volatility of simulated exchange rates, including high consumer risk aversion (CKM), sticky prices and/or wages (although duration of price contracts required to generate enough exchange rate volatility seemed to be improbably high⁴), low elasticity of substitution between imports and exports (Corsetti, Dedola and Leduc, 2008; Tille, 2001), and information frictions (Martínez-García, 2010). Matching exchange rate persistence has been a more difficult issue to resolve; current research in this area suggests increasing interest rate inertia in the monetary policy rule, introducing asymmetries in the duration of nominal rigidities (both in Benigno, 2004), modeling different exchange rate regimes - flexible, managed or fixed (Benigno and Benigno, 2008), and adding real shocks to the models (Steinsson,

²For recent surveys on the literature which highlight the dynamics of real exchange rates characterized as a nonlinear process, see Taylor and Taylor (2004), Taylor (2006) and Ahmad and Glosser (2011). Engel, Mark and West (2007) provide a thorough overview of the developments in both theoretical and empirical literature.

³The only exception we are aware of is Corsetti, Dedola and Leduc (2008), who estimate exchange rate pass-through coefficients using simulated data.

⁴Bergin and Feenstra (2001) find that price contracts that last for 12 quarters are still not enough to generate the observed persistence of the real exchange rates. Kollmann (2001) notes that the nominal contract duration should be set at 15 quarters to match the data.

2008). However, to our knowledge, a formal side-by-side evaluation of these extensions is lacking.

Recent empirical studies, which have focused on estimating the dynamics of real exchange rates, have found evidence of nonlinearities in its behavior. The motivation behind this view, which began to gather support in the late 1990's, arose from the failures of a linear framework: assuming linear dynamics and perfect arbitrage implies that the speed of adjustment is constant at all levels of deviations from PPP. However, several reasons have been put forth which would indicate that perfect arbitrage is not observed in the real world, and consequently nonlinearities would exist in the dynamics of real exchange rates. These reasons range from transactions costs (Michael, Nobay and Peel, 1997; Obstfeld and Taylor, 1997) and heterogeneity of agent's beliefs (Kilian and Taylor, 2003), to misalignments in the foreign exchange market resulting in a lack of co-ordination (Sarno and Taylor, 2001; Reitz and Taylor, 2008). These papers are among many which have found that the data on real exchange rates can be parsimoniously characterized as smooth transition autoregressive (STAR) process, or one of its variants.⁵ However, these studies lack rigorous theoretical foundations that could help to explain the existence of nonlinearities in real exchange rate behavior.

In this paper, we build a two-country DSGE model that includes a wide array of frictions commonly used in the theoretical literature to answer two sets of questions, which we hope will offer a better understanding of the dynamics of the simulated real exchange rates in the DSGE framework. First, we study their persistence and volatility as a function of the model structure. We make a small contribution here to the extant literature by incorporating potential solutions that have been suggested in response to CKM, and report the extent to which those solutions resolve the issue. Our main contribution arises from the second question: do the nonlinearities that exist within the structural DSGE model translate into the type of nonlinear dynamics observed for real exchange

⁵A small sample of papers include Sarantis (1999), Baum et al (2001), Taylor et al (2001), and Paya and Peel (2006).

rates in the time series literature? The particular type of nonlinearity that we consider is the STAR process, given its success within the empirical literature.

Our main findings are as follows. First, the solutions that have been put forth in response to the issues brought out by CKM do a good job in matching the business cycle properties of real exchange rates when both data and simulated series are HP filtered. However, we find that only a couple of model specifications come close to matching key properties (volatility, persistence and nonlinearity) of unfiltered data, which is primarily used in time series estimation. More specifically, home bias in consumption, as well as the combination of pricing to market, home bias and non-traded goods, approaches the levels of persistence we observe in the data, although none of the specifications examined produce the observed real exchange rate volatility. When evaluating the incidence of nonlinearity that is detected in the simulated real exchange rate series, we find that home bias yields the highest incidence amongst the four basic specifications we consider, and that the combination of local currency pricing, home bias and non-traded goods yields the highest incidence of nonlinear dynamics among all the specifications examined. These results are robust to the choice of solution method (first or second order approximation around the model's steady state), although persistence, volatility and the incidence of nonlinearity increase slightly with the order of approximation.

Additionally, we investigate the strength of the mechanisms that generate nonlinear real exchange rate dynamics in the two most promising model extensions: home bias and non-traded goods. The results of our simulations indicate that the incidence of nonlinearity is increasing in the degree of home bias. Moreover, in the non-traded goods specification, the incidence of nonlinearity is symmetric (and largest) about the point where the relative productivity shocks in the non-tradable and tradable sectors are equal. Finally, in both model specifications the dynamics of real exchange

rates are best characterized as an exponential smooth transition autoregressive process, indicating symmetric adjustment to the PPP norm.

The remainder of the paper is structured as follows: the basic model, its extensions and calibration are outlined in Section 2; Section 3 discusses results of the estimation of the simulated time series and presents the main results; Section 4 further investigates the key channels that appear to generate nonlinear dynamics; finally, Section 5 concludes.

2 The Model

Since the mid-1990's, DSGE models have rapidly gained popularity and are now very much in vogue. Used by researchers and policymakers alike, they contain a variety of features that have both theoretical appeal and allow for the examination of potentially any policy question. Although DSGE models may incorporate a variety of specific features designed to elucidate the particular problem being studied, they also contain a lot of common structural elements. The class of New Open Economy (NOEM) models considered in this paper is characterized by optimizing behavior on the part of agents, rational expectations and nominal frictions in the form of wage and price inertia that allow monetary policy to influence real variables.

Within the NOEM framework, real exchange rates are typically determined through the PPP equation as a function of nominal exchange rates and international relative prices. In order to generate dynamic behavior of real exchange rates, we must specify the source(s) of deviations from the purchasing power parity. The recent NOEM literature has identified several of these⁶: presence of non-traded goods in the consumer basket (Corsetti, Dedola and Leduc, 2008), home bias in consumer preferences (CKM; Faia and Monacelli, 2008; Steinsson, 2008), sticky prices (CKM;

⁶The following references comprise a non-exhaustive list of papers that have used one or more of these features.

Bergin and Feenstra, 2001; Kollmann, 2001) and local currency pricing (CKM; Corsetti, Dedola and Leduc, 2008; Benigno, 2004; Steinsson, 2008). We add these features (and highlight the mechanisms they add to real exchange rate dynamics) one at a time to an otherwise basic two-country DSGE model and study the resulting time-series properties of the simulated real exchange rate. We also consider the case of incomplete international asset markets, in which the uncovered interest parity (UIP) condition does not hold, and nominal (and real) exchange rates become decoupled from the relative interest rate movements.

The choice of shocks has non-trivial consequences for the behavior of exchange rates. Monetary policy, which requires the presence of nominal rigidities to affect real variables, must itself be quite persistent to generate the observed half-lives of real exchange rates. On the other hand, real shocks, which can include (but are not limited to) government spending, technology, and demand for home goods, have been found to generate hump-shaped response of exchange rates, which we see in the data (Steinsson, 2008). We include all of the disturbances above in our model to increase the chances of matching simulated time series to the data.

The benchmark model is composed of two countries, *Home* (H) and *Foreign* (F). Each country is populated by infinitely lived households of measure one; there is no migration. Households consume all varieties of home and foreign goods; international asset markets are complete. Each country has measure one of firms that use country-specific labor and capital to produce a continuum of domestic goods varieties. Firms are monopolistically competitive, and the prices they set for their products are sticky à la Calvo (1983).

As a matter of notation, subscripts H and F will refer to a good's country of origin; asterisks will indicate that it is consumed in country F . For example, C_H^* denotes consumption of country H 's good in country F . The two economies have a similar structure; therefore, most of the equations

will be presented only for the Home country.

2.1 The Benchmark Specification

The only potential source of real exchange rate fluctuations in the benchmark model is nominal rigidity. Since the setup of the benchmark model is well documented in the literature, we keep our description of it short.

2.1.1 Firms

Each country has a continuum of firms that produce tradable goods indexed by f on the unit interval. At time t , each Home firm rents capital $K_{T,t-1}(f)$ from the domestic households at the rate R_t , hires a labor bundle $N_{T,t}(f)$ at the rate W_t and produces one of the varieties of the domestic good. Firms use the following CRS technology to produce output:

$$Y_{H,t}(f) = A_{T,t} [K_{T,t-1}(f)]^\nu [N_{T,t}(f)]^{1-\nu},$$

where $0 < \nu < 1$, and $A_{T,t}$ denotes the level of productivity enjoyed by all the home firms at time t .⁷

All goods varieties are then bundled into a composite home good using the Dixit-Stiglitz aggregator $Y_{H,t} = \left\{ \int_0^1 [Y_{H,t}(f)]^{\frac{\sigma_T-1}{\sigma_T}} df \right\}^{\frac{\sigma_T}{\sigma_T-1}}$, where $\sigma_T > 1$. This composite good can then be used for public and private consumption or private investment.

As in Calvo (1983), home and foreign firms reset their prices each period with a constant probability $(1 - \alpha)$ and $(1 - \alpha^*)$, respectively; otherwise, the old prices remain in effect. If a (home) firm f gets to announce a new price in period t , it chooses $\tilde{P}_{H,t}(f)$ to maximize its expected discounted

⁷We use the subscript T to differentiate between tradable and non-tradable goods (introduced in Section 2.2).

future profits

$$E_t \sum_{j=t}^{\infty} \Omega_{t,j} \alpha^{j-t} \left\{ \tilde{P}_{H,t}(f) \left[Y_{H,j}^d(f) + Y_{H,j}^{*d}(f) \right] - TC_j \left(Y_{H,j}^d(f) + Y_{H,j}^{*d}(f) \right) \right\}$$

where profits are a function of total aggregate demand and TC represents total costs. Here $\Omega_{t,j}$ captures the stochastic discount factor of households, and the two terms in the brackets denote the aggregate demands for home goods bundle from home and foreign households, investors and governments, respectively. Notice that in equilibrium, $Y_{H,j}^d(f) + Y_{H,j}^{*d}(f) = Y_{H,t}(f)$.

Given the price-setting behavior of individual firms, the aggregate price index of the Home goods can be written as

$$P_{H,t}^{1-\sigma_T} = (1-\alpha) \tilde{P}_{H,t}^{1-\sigma_T} + \alpha P_{H,t-1}^{1-\sigma_T}$$

2.1.2 Households

There is a continuum of households in the home country, indexed by h on the unit interval. A representative household maximizes expected lifetime utility

$$U_t(h) = E_t \sum_{j=t}^{\infty} \beta^{j-t} \left\{ \frac{[C_{T,j}(h)]^{1-\Theta}}{1-\Theta} - \frac{[L_{T,j}(h)]^{1+\chi}}{1+\chi} \right\}$$

Here $C_{T,t}(h)$ denotes the household's consumption of the composite good, which is aggregated from home and foreign goods using the CES aggregator:

$$C_{T,t}(h) = \left[\gamma^{\frac{1}{\theta}} [C_{H,t}(h)]^{\frac{\theta-1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}} [C_{F,t}(h)]^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \quad (1)$$

where θ measures the elasticity of substitution between home and foreign goods. In the benchmark specification, we set $\gamma = \gamma^* = \frac{1}{2}$, so that the consumption baskets of home and foreign households are identical.

Since the Law of One Price (LOP) clearly holds in this specification, the prices of the two composite

goods, which also represent the countries' CPIs, are given by

$$P_{T,t} = \left[\gamma P_{H,t}^{1-\theta} + (1-\gamma) S_t P_{F,t}^{*(1-\theta)} \right]^{\frac{1}{1-\theta}} \quad (2a)$$

$$P_{T,t}^* = \left[\gamma^* P_{F,t}^{*(1-\theta)} + (1-\gamma^*) S_t^{-1} P_{H,t}^{(1-\theta)} \right]^{\frac{1}{1-\theta}} \quad (2b)$$

Here S_t is the nominal exchange rate, expressed in units of Home currency per one unit of Foreign currency.

Households supply differentiated labor services to all the firms in their country. The composite labor bundle used in production by any given home firm can be written as: $N_{T,t}(f) = \left\{ \int_0^1 [L_{T,t}(h)]^{\frac{\phi-1}{\phi}} dh \right\}^{\frac{\phi}{\phi-1}}$. Each household enjoys a degree of monopolistic power in setting its wage $W_t(h)$. Wages are sticky, and in any given period a household gets to reset its wage with probability $(1-\omega)$ ($(1-\omega^*)$ abroad).

Households in both countries have access to a complete contingent claims market. Each home household faces the following budget constraint:

$$\begin{aligned} E_t[\Delta_{t,t+1} D_t(h)] + P_{T,t} [C_{T,t}(h) + I_{T,t}(h) - T_t(h)] = \\ = W_t(h) L_{T,t}(h) + D_{t-1}(h) + R_t K_{T,t-1}(h) + \Pi_{T,t}(h) \end{aligned}$$

The first term on the left-hand side is the price of a portfolio of state-contingent bonds traded domestically, and $D_{t-1}(h)$ is the payoff of such portfolio in period t . Households receive transfers $P_t T_t(h)$ from their government (which can be negative in the event of lump-sum taxation). $\Pi_{T,t}(h)$ represents profits of all domestic firms.

The household's capital accumulation is given by

$$K_{T,t}(h) = (1-\delta)K_{T,t-1}(h) + I_{T,t}(h) - \frac{1}{2}\psi \left[\frac{I_{T,t}(h)}{K_{T,t-1}(h)} - \delta \right]^2 K_{T,t-1}(h)$$

The investment good $I_{T,t}(h)$ has the same composition as consumption in (1).

Households maximize utility subject to the budget, labor demand and capital accumulation constraints by choosing wage rate $W_t(h)$, consumption $C_{T,t}(h)$, portfolio holdings $D_t(h)$ and investment $I_{T,t}(h)$.

2.1.3 The Government

The Home country monetary policy that can be described by a variant of the Taylor Rule, in which the short-term nominal interest rate responds (with inertia) to deviations of inflation and output from their targets:

$$i_t = (1 - \rho_i) \bar{i} + \rho_i i_{t-1} + (1 - \rho_i) [\rho_\pi \pi_t + \rho_y \Delta y_t] + \varepsilon_{i,t} \quad (3)$$

Here $\bar{i} = \frac{1}{\beta} - 1$ is the steady state level of the interest rate, π_t is the growth rate of CPI, and Δy_t denotes output gap, which we measure as the deviation of output from its steady state level⁸. The Foreign economy pursues independent monetary policy analogous to rule (3).

We let government purchases be described by an autoregressive process

$$\ln G_{T,t} = (1 - \rho_g) \ln \overline{G_T} + \rho_g \ln G_{T,t-1} + \varepsilon_{g,t},$$

where $\varepsilon_{g,t}$ is a white noise process and $\overline{G_T}$ denotes the steady state level of government spending; $G_{T,t}$ has the same composition as consumption in (1). Both governments balance their budgets every period, so $G_{T,t} = T_t$ and $G_{T,t}^* = T_t^*$.

2.1.4 Aggregation and Equilibrium

The following market clearing conditions complete the description of the model.

⁸Generally, output gap can be modeled as deviation of current output from either (a) its steady state level; (b) flexible-price level; or (c) last period level.

$$[C_{H,t} + I_{H,t} + G_{H,t}] + [C_{H,t}^* + I_{H,t}^* + G_{H,t}^*] = [Y_{H,t} + Y_{H,t}^*] DP_{H,t} \quad (4)$$

$$[C_{F,t} + I_{F,t} + G_{F,t}] + [C_{F,t}^* + I_{F,t}^* + G_{F,t}^*] = [Y_{F,t} + Y_{F,t}^*] DP_{F,t}^* \quad (5)$$

$$\int_0^1 D_t(h) dh + \int_0^1 D_t^*(h) dh = 0 \quad (6)$$

The terms $DP_{H,t}$ and $DP_{F,t}^*$, defined in Appendix A, describe the dispersion of firm-level prices from the aggregate.

Equilibrium in the economy is defined by (4)-(6) and the first order conditions of the agents (listed in Appendix A), given the form of monetary and fiscal policy rules described above.

2.2 Extensions

The benchmark specification, in which PPP holds at all times, generates virtually no real exchange rate movement and therefore serves as a stepping stone to richer frameworks with more frictions. Below we outline four widely used extensions of the basic DSGE model.

2.2.1 Home Bias in Consumption

The existence of home bias in consumption has been well documented in the trade literature (for example, in Obstfeld and Rogoff, 2000) and has been found to have important implications on the optimal monetary policy conduct, as mentioned in the introduction.

To introduce home bias in our model, we let the parameters γ and γ^* in (1) differ from their benchmark value of $\frac{1}{2}$. By allowing for a different composition of Home and Foreign consumption baskets, we decouple movements in the two countries' CPI levels (described by equations (2)), which breaks the absolute PPP relationship and thus introduces richer dynamics of the real exchange rate.

To see this, we substitute (2) into the definition of the real exchange rate $Q_t \equiv \frac{S_t P_{T,t}^*}{P_{T,t}}$ and linearize around the symmetric steady state:

$$q_t = [\gamma^* + \gamma - 1] \tau_t$$

where lowercase letters indicate log deviations from steady state, and $\tau_t \equiv p_{F,t}^* + s_t - p_{H,t}$ represents the terms of trade.

In the case of symmetric home bias ($\gamma = \gamma^* = \frac{1}{2}$ or, more generally, $\gamma = 1 - \gamma^*$), movements in prices of tradables do not have a net effect on real exchange rate; as the economies move away from the symmetric case, real exchange rate movements grow in magnitude in response to fluctuations in terms of trade.

2.2.2 Local Currency Pricing (LCP)

Empirical evidence points to a rather low degree of pass-through from exchange rates to import prices, which of course is inconsistent with our benchmark assumption of the LOP on an individual, as well as the aggregate price level.⁹ The literature has identified several potential causes of this phenomenon: local currency pricing by firms with some degree of monopoly power, the presence of non-traded goods either in the product distribution network or directly in the consumer basket, or price stickiness at the consumer level. In fact, local currency pricing is one of three key factors, identified by Devereux and Engel (2002) in their investigation of the ‘exchange rate disconnect’ puzzle, that they say is necessary for generating volatile exchange rates.

We begin by adding international price discrimination by firms, which may optimally choose to charge different prices at home and abroad, depending on their expectations of the future nominal

⁹For example, Engel (1993) finds empirical evidence that the volatility of the price of a good relative to a similar good within a country, is lower than the volatility of the price of a good, relative to the price of the same good in a different country; also Engel and Rogers (1996) find that the "border effect" introduces significant variation in the price of a good sold in different countries.

exchange rates. When a (home) firm f gets to announce a new price in period t , it chooses $\tilde{P}_{H,t}(f)$ and $\tilde{P}_{H,t}^*(f)$ to maximize its expected discounted future profits

$$E_t \sum_{j=t}^{\infty} \Omega_{t,j} \alpha^{j-t} \left\{ \tilde{P}_{H,t}(f) Y_{H,j}^d(f) + S_j \tilde{P}_{H,t}^*(f) Y_{H,j}^{*d}(f) - TC_j \left[Y_{H,j}^d(f) + Y_{H,j}^{*d}(f) \right] \right\}$$

The presence of price rigidity implies that home firms optimally take into account the entire path of future expected nominal exchange rates S_t when setting foreign prices for their products; thus, the LOP need not hold:

$$\begin{aligned} \tilde{p}_{H,t} &= (1 - \alpha\beta)mc_t + \alpha\beta E_t(\tilde{p}_{H,t+1}) \\ \tilde{p}_{H,t}^* &= (1 - \alpha\beta)[mc_t - s_t] + \alpha\beta E_t(\tilde{p}_{H,t+1}^*) \end{aligned}$$

where mc_t denotes the marginal cost of production. The aggregate price indices of the Home goods at home and abroad are now given by

$$P_{H,t}^{1-\sigma_T} = (1 - \alpha)\tilde{P}_{H,t}^{1-\sigma_T} + \alpha P_{H,t-1}^{1-\sigma_T} \quad (7a)$$

$$P_{H,t}^{*(1-\sigma_T)} = (1 - \alpha)\tilde{P}_{H,t}^{*(1-\sigma_T)} + \alpha P_{H,t-1}^{*(1-\sigma_T)} \quad (7b)$$

In addition to the home bias mechanism described above, the real exchange rate now responds to international price differences of the same good. Linearizing the definition of the real exchange rate around the steady state yields

$$q_t = [\gamma^* + \gamma - 1] \tau_t + \gamma \varepsilon_t^H + \gamma^* \varepsilon_t^F,$$

where ε_t^H and ε_t^F measure deviations from the LOP that arise due to local currency pricing coupled with nominal rigidities: $\varepsilon_t^F \equiv p_{F,t}^* + s_t - p_{F,t}$ and $\varepsilon_t^H \equiv p_{H,t}^* + s_t - p_{H,t}$.

2.2.3 Non-Traded Goods

The second way of lowering the degree of pass-through from exchange rates to inflation is to introduce non-traded goods. To do so, we add a continuum of firms, indexed by $n \in [0, 1]$, which,

similar to the producers of the tradables above, hire capital and labor from local households to produce one of the varieties of non-tradable goods:

$$Y_{N,t}(n) = A_{N,t} [K_{N,t-1}(n)]^\zeta [N_{N,t}(n)]^{1-\zeta}$$

We assume that there are no correlations in the productivity processes across sectors. All the varieties of non-traded goods are bundled together analogously to the tradable output: $Y_{N,t} = \left\{ \int_0^1 [Y_{N,t}(n)]^{\frac{\sigma_N-1}{\sigma_N}} dn \right\}^{\frac{\sigma_N}{\sigma_N-1}}$. Each firm in the non-traded sector chooses a price $\tilde{P}_{N,t}^{1-\sigma_N}(n)$ to maximize profits; this price can be reset each period with a probability $(1-\alpha)$. The price of the non-traded bundle is therefore given by $P_{N,t}^{1-\sigma_N} = (1-\alpha)\tilde{P}_{N,t}^{1-\sigma_N} + \alpha P_{N,t-1}^{1-\sigma_N}$.

The household's problem is now updated to include consumption of non-traded goods:

$$U_t(h) = E_t \sum_{j=t}^{\infty} \beta^{j-t} \left\{ \frac{[C_j(h)]^{1-\Theta}}{1-\Theta} - \frac{[L_j(h)]^{1+\chi}}{1+\chi} \right\},$$

where $C_t(h)$ is given by

$$C_t(h) = \left[\mu^{\frac{1}{\eta}} [C_{N,t}(h)]^{\frac{\eta-1}{\eta}} + (1-\mu)^{\frac{1}{\eta}} [C_{T,t}(h)]^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \quad (8)$$

Here η denotes the elasticity of substitution between tradables and non-tradables, and $0 < \mu < 1$ determines the relative weight of non-tradables in consumption. Investment used to augment capital stock, as well as government purchases, are now also composed of traded and non-traded goods according to (8). The new CPI level is described by

$$P_t = \left[\mu P_{N,t}^{1-\eta} + (1-\mu) P_{T,t}^{1-\eta} \right]^{\frac{1}{1-\eta}} \quad (9)$$

The definitions of $C_{T,t}(h)$ and $P_{T,t}$ are still given by equations (1) and (2a). Finally, the profits of the non-tradable sector are added to the household budget constraint.

To close the model, we have to add the following market clearing conditions:

$$DP_{N,t}Y_{N,t} = C_{N,t} + I_{N,t} + G_{N,t}$$

$$L_t = N_{N,t} + N_{T,t}$$

$$K_t = K_{N,t} + K_{T,t},$$

plus analogous equations for the foreign economy.

It is clear from (9) that, even in the absence of home bias, local currency pricing or nominal rigidities (in which case the composition and the price of the tradable baskets equalize), relative technological growth between traded and non-traded sectors can, through the Harrod (1933), Balassa (1964) - Samuelson (1964) effect, lead to movements in the real exchange rate.

2.2.4 Incomplete Markets

While the existence of the internationally traded bond simplifies aggregation by equalizing marginal utilities of wealth in the two countries, it has an uncomfortable implication that the variation in the relative consumption growth rates is directly linked to real exchange rate movements. By combining international risk-sharing conditions and households' first order conditions we get $\Delta q_{t+1} = \Theta [\Delta c_{t+1} - \Delta c_{t+1}^*]$. In the data, the correlation between these two variables is 0.07, which is rather low. This observation motivates an examination of the incomplete markets specification, which has the potential to lower the theoretical correlation to the level observed in the data.

We continue assuming (for model tractability) that all consumers can perfectly share risks within a country; a risk-free bond A_t , issued by the foreign country, can be traded internationally. Home households can buy the bond at a higher price than foreign households. Home consumers' price $S_t [1 + i_t^* - p(a_t - \bar{a})^2]^{-1}$ depends on their position in the international asset market. Here a_t represents the net foreign wealth of home consumers, \bar{a} is the steady state level of a_t , and the

parameter p captures transaction costs¹⁰. As lenders ($a_t > 0$), domestic households pay a higher price for the bond, and as borrowers they must offer a rate of return higher than $(1 + i_t^*)$. A home household's budget constraint becomes

$$\begin{aligned} & E_t[\Delta_{t,t+1} D_t(h)] + S_t A_t(h) + P_t [C_t(h) + I_t(h) - T_t(h)] = \\ & = W_t(h) L_t(h) + D_{t-1}(h) + \left[1 + i_t^* - p(a_t - \bar{a})^2\right] S_t A_{t-1}(h) + R_t K_{t-1}(h) + \Pi_{T,t}(h) + \Pi_{N,t}(h) \end{aligned}$$

The asset market clearing conditions are modified as follows:

$$\begin{aligned} \int_0^M D_t(h) dh &= 0 \\ \int_0^{M^*} D_t^*(h) dh &= 0 \\ \int_0^M A_t(h) dh + \int_0^{M^*} A_t^*(h) dh &= 0 \end{aligned}$$

2.2.5 Sources of Real Exchange Rate Movement

Before moving on to the results of the model simulations, it is perhaps useful to summarize the various mechanisms that can contribute to the dynamics of the real exchange rate. To this end, equation (10) presents the disaggregated definition of the real exchange rate in the full model (including home bias, non-traded goods, local currency pricing and nominal rigidities):

$$Q_t = S_t \frac{\left\{ \mu P_{N,t}^{1-\eta} + (1 - \mu) \left[\gamma P_{H,t}^{1-\theta} + (1 - \gamma) P_{F,t}^{1-\theta} \right]^{\frac{1-\eta}{1-\theta}} \right\}^{\frac{1}{1-\eta}}}{\left\{ \mu^* \left(P_{N,t}^* \right)^{1-\eta} + (1 - \mu^*) \left[\gamma^* \left(P_{F,t}^* \right)^{1-\theta} + (1 - \gamma^*) \left(P_{H,t}^* \right)^{1-\theta} \right]^{\frac{1-\eta}{1-\theta}} \right\}^{\frac{1}{1-\eta}}} \quad (10)$$

where the prices of tradables evolve according to (7) and its foreign economy equivalent. The real exchange rate fluctuates due to (a) movements of the nominal exchange rate S_t ; (b) presence

¹⁰Equilibrium dynamics of a small open economy with incomplete asset markets generally include a random walk component; the transaction cost modification guarantees stationarity of the model. See Schmitt-Grohé and Uribe (2003) for explicit treatment of the problem.

of non-traded goods ($\mu > 0$); (c) differences in the composition of the tradable baskets ($\gamma \neq 1 - \gamma^*$); (d) existence of firm market power ($P_{H,t} \neq Q_t P_{H,t}^*$); and (e) nominal rigidities ($\alpha \neq 0$). Finally, the incomplete markets specification breaks the UIP relationship between real interest rates and exchange rates and also decouples the movements of the latter from the dynamics of relative consumptions.

It is also useful to note that equation (10) clearly demonstrates the existence of structural nonlinearities for real exchange rates arising from the microfoundations within the DSGE framework. One question that we address in section 3 is whether these micro-level nonlinearities translate into the type of macro-level nonlinearities that are able to be identified within the time series framework. If the answer is yes, that micro-level nonlinearities *do* translate to macro-level nonlinearities, and are able to be identified by time series models, then it provides an external validity to both literatures: DSGE models and the dynamics of real exchange rates represented by time series models.

2.3 Calibration

Since the goal of our paper is to understand how standard DSGE models fare in reproducing the observed non-linearities in the behavior of real exchange rates, we follow CKM in parameterizing our model. Table 1 lists the parameter values; most of them are quite common in the literature, while a few others merit further discussion. Each time period in the model corresponds to one quarter.

The high value of the risk aversion parameter is motivated by the need to replicate high observed volatility of real exchange rates. When asset markets are complete, real exchange rate is a function of relative consumptions: $Q_t = \Gamma \left[\frac{C_t}{C_t^*} \right]^\Theta$. Given the U.S. and UK consumption volatility, setting $\Theta = 5$ matches the volatility of Q_t to that in the data.

Productivity in the two countries evolves according to the following autoregressive process:

$$\begin{bmatrix} \ln A_{i,t} \\ \ln A_{i,t}^* \end{bmatrix} = \begin{bmatrix} A_{11}^i & A_{12}^i \\ A_{21}^i & A_{22}^i \end{bmatrix} \begin{bmatrix} \ln A_{i,t-1} \\ \ln A_{i,t-1}^* \end{bmatrix} + \begin{bmatrix} \varepsilon_{A,t}^i \\ \varepsilon_{A,t}^{*i} \end{bmatrix}, \quad \text{for } i \in \{T, N\}$$

There are no spillovers or cross-correlations between the two sectors.

We follow Corsetti, Dedola and Leduc (2008) and the sources referenced therein to calibrate the non-traded sector. We set the elasticity of substitution between tradables and non-tradables, η , to 0.74; assume that the share of non-tradables in consumption is 60%, and the share of capital in the production of non-tradables is $\zeta = 0.44$.

The value of the capital adjustment cost parameter ψ is calibrated to replicate the ratio of standard deviations of consumption to output. Based on U.S. data, we set the ratio of government debt to GDP to 20 percent, and let $\rho_g = 0.97$.

Following CKM, the technology process parameters are given by $Var(\varepsilon_A^T) = 0.007^2$ and $Cov(\varepsilon_A^T, \varepsilon_A^{*T}) = 1.2 \times 10^{-5}$. For simplicity, we assume that the shocks in the non-traded sector have the same covariance matrix as those in the traded sector. Government spending shock variance is $Var(\varepsilon_g) = 0.0001$. Finally, we calibrate $Var(\varepsilon_i) = Var(\varepsilon_i^*)$ to match the volatility of output, and $Cov(\varepsilon_i, \varepsilon_i^*)$ to match the cross-correlation of U.S. and UK outputs.

Solution to the model is found using perturbation methods described in Schmitt-Grohé and Uribe (2004) and Collard and Juillard (2001); computer code is written in Dynare (Collard and Juillard, 2003).

3 Results

3.1 Business Cycle Properties

For easier comparison with the existing theoretical literature, Table 2 lists several relevant moments of the simulated data. In addition to examining the benchmark model and its four extensions, we also study several combinations of the structural primitives. Thus, column 8 refers to the specification of the model that includes both LCP and home bias (we dub this version CKM since Chari, Kehoe and McGrattan (2002) used it as the starting point in their analysis). Columns 8 and 9 add incomplete markets (CKM_IM) and (separately) non-traded goods (CKM_NT) to the CKM model.

The interaction between nominal rigidities and LCP (column 5), which limits the pass-through of exchange rate movements to national prices, produces much more volatile real exchange rates compared to other basic model extensions, although it still falls short of the volatility in the data. We find that the combination of LCP and home bias (CKM, column 8) produces the closest match of real exchange rate volatility, albeit on the high side, whereas adding non-traded goods to the mix (CKM_NT, column 10) overshoots the volatility by over 30%. Our results suggest that DSGE models are very successful in matching this particular property of real exchange rates.

Turning to persistence, we find that home bias and non-traded goods are the likeliest specifications to match the empirical autocorrelation of the real exchange rates, although the former undershoots it and the latter generates too much persistence.

As is commonly found in DSGE literature, other moments of real exchange rates are less satisfactory. For example, all versions of the model overstate the international consumption correlation, although the CKM_IM specification comes fairly close to matching the 0.43 value observed in the data.

Moreover, none of the models studied in this paper are able to replicate the empirical correlation between GDP and the real exchange rate.

Overall, we conclude that the DSGE framework (outfitted with certain structural features like local currency pricing or non-traded goods) is capable of replicating the observed real exchange rate volatility and persistence, in terms of their business cycle properties, although more work needs to be done to bring other moments of the models in line with the data.

3.2 Estimation of the Simulated Time Series

We now turn our attention to the time series properties of the real exchange rates simulated using the different models outlined in Section 2. For each version of the model, we generate 1000 trials (each 5000 observations long) of the (log) real exchange rate and then examine the properties of the simulated data in order to infer the underlying dynamics embedded within the series. For each trial, we assume that the simulated data may be succinctly captured by a univariate autoregressive process of the form

$$q_t = \lambda_1 q_{t-1} + \dots + \lambda_p q_{t-p} + \varepsilon_t, \quad (11)$$

where ε_t is white noise. The simulated series is stationary if the sum of the autoregressive coefficients, $|\sum_{i=1}^p \lambda_i|$, is less than 1.¹¹ Re-writing (11) into an augmented Dickey-Fuller regression form allows us to estimate a measure of the persistence of the real exchange rate. In this form, (11) can be written as:

$$q_t = \Lambda q_{t-1} + \varphi_1 \Delta q_{t-1} + \dots + \varphi_{p-1} \Delta q_{t-p+1} + \varepsilon_t \quad (12)$$

Consequently, the dynamics of the real exchange rate are governed by the key parameters $\{\lambda_1, \dots, \lambda_p\}$, since $\varphi_j = -(\lambda_{j+1} + \dots + \lambda_p)$ for $j = 1, \dots, p-1$ and $\Lambda \equiv \sum_{j=1}^p \lambda_j$. Λ represents our measure of

¹¹As a precursor to our analysis, we run unit root tests on the simulated data to verify that the real exchange rate is stationary. In all the trials, we are able to reject the null of nonstationarity for the series.

persistence of the underlying data generating process.¹²

It should be noted that since the time series literature examining the dynamics of real exchange rates utilizes raw (unfiltered) data, we proceed to examine the dynamics of the simulated data without HP filtering it. The properties of the actual data are summarized in Table 3 for the various G7 countries in our sample.

For the 1970Q1 - 2010Q1 period, we are unable to reject a null hypothesis of a unit root in the real exchange rate for the majority of the countries. Although this might be a point of concern since the DSGE models above were found to generate stationary real exchange rates, it is useful to note that other studies were also unable to reject a unit root in post-Bretton Woods data: see, for example, Lothian and Taylor (1996) and Ahmad and Craighead (2011). This result is unsurprising given the notoriety of low power of unit root tests. In fact several papers have found that once longer spanning data is utilized, the behavior of the real exchange rate appears to be stationary (Lothian and Taylor, 1996 and 2008; Ahmad and Craighead, 2011). Additionally, when fitting an autoregressive structure to the real exchange rate, we find that Λ , the sum of the autoregressive coefficients, is less than one in absolute value, even though the coefficient on the AR(1) term is larger than one for Canada, Japan and the UK.

3.3 Persistence and Volatility

The first step in our analysis involves fitting the univariate autoregressive process to both the actual as well as the simulated data, and this requires selecting an order of autoregression, p . We use the SBC information criterion as a basis for selecting p , given that the SBC is known to favor parsimony

¹²Other measures of persistence that have been proposed by various authors include the computation of a half-life from a simple AR(1) coefficient or via the decay pattern from an impulse response function (e.g., Murray and Papell, 2002; Dias and Marques, 2010), the largest autoregressive root (Stock, 1991), as well as other measures that incorporate a bias correction for the half life (for example, Rossi, 2005).

over other information criteria (for example, Akaike) or other selection methods like examining the partial autocorrelation function. We allow the data to select the order of autoregression for the each trial in the simulation, and then compute the Ljung-Box (1979) Q statistics to ensure that our model has adequate fit. If the null of no serial correlation in the residuals is rejected, we increase p , re-estimate equation (11) and repeat the test until we are unable to reject the null. Our choices of the autoregression order are presented in Tables 3 through 5 for both the actual data and the DSGE-simulated real exchange rates.

We find that $p \in \{1, 2\}$ fits the actual data (Table 3). When considering the simulated data (panel A of Table 4), the median value of p selected using the algorithm above is also either 1 or 2 across the 1000 trials. The table also reports the 5th and 95th quantiles from the distribution of p ; our results indicate that it is the tightest in the LCP specification. The distributions of p in the other versions of the model display a greater degree of dispersion, but not overly so, with the largest amount of dispersion occurring in the home bias and the CKM_NT cases. Our findings informally suggest that all of the studied specifications of the theoretical model match the data we examine in Table 3 along this particular dimension.

The more important question concerns persistence and volatility of the simulated series (that have been difficult to replicate in theoretical models), which we also report in panel A of Table 4. In the data, the average volatility of the log real exchange rate series across the various countries is 0.17. As is readily apparent in Table 4, none of the versions of the model approach that degree of volatility, despite the fact that the CKM specification was able to match the volatility from the business cycle perspective when the data and the simulated series were HP filtered (see Table 2).¹³

The version of the model which most closely resembles the data (at least for the US-Canadian

¹³Although we report the mean volatility in the table, the median volatility is approximately the same, at least to three decimal places.

exchange rate) combines local currency pricing, home bias and non-traded goods (CKM_NT), although the distribution of volatilities generated across the thousand trials still falls short of the empirical findings. This result is somewhat surprising, given that the non-traded goods version by itself generates very little volatility within the DSGE framework. Clearly, the interaction of individual extensions contained in CKM_NT yields a more volatile real exchange rate than in any of the underlying specifications (pricing-to-market, home bias and non-traded goods in this case).

Turning to real exchange rate persistence, we obtain more encouraging results when estimating the value of Λ using (12). We find that the version of the model with only non-traded goods as well as the version with incomplete markets are able to generate sufficiently persistent real exchange rates, comparable to those found in the data (the average measure of real exchange rate persistence across the different countries is 0.947), and that the home bias specification falls only marginally short of the empirical estimates. Once again, surprisingly, the CKM specification (incorporating both local currency pricing and home bias) does not generate sufficiently persistent real exchange rates. It is only when non-traded goods are added to the mix that we obtain persistence in the neighborhood of what is seen in the data. Interestingly, the version of CKM that also incorporates incomplete markets does not perform nearly as well as either of its basic components (LCP, home bias or incomplete markets). Once again, we conclude that the interactions of individual building blocks of our models produce patterns in real exchange rate dynamics that are quite distinct from those in the building blocks themselves.

Some intuition for the persistence result can be gained by examining the distributions of AR(1) coefficients generated across the different trials, which are plotted in Figures 1 and 2. The median of the distribution lies significantly below our point estimates in Table 3 for the actual data, and as such it is highly suggestive that the low estimates of the AR(1) parameter are the key determinant

in driving the estimated persistence parameter Λ downwards.

One problem with the estimates reported in Tables 3 and 4, as Andrews (1993) points out, is the fact that the least squares estimates of the persistence parameter in autoregressive models are biased downwards. Andrews (1993) proposes an exact bias correction method for an AR(1) model, while Andrews and Chen (1994) outline a bias correction method in the AR(p) case. We utilize both of these methods to obtain a median unbiased estimate for the true value of the persistence parameter. In the case when p is one, the bias correction procedure is exact, while for $p > 1$, it is an approximation to the true unbiased estimate. Moreover, the bias correction methods proposed by Andrews (1993) and Andrews and Chen (1994) are robust to the presence of a unit root in the series. We calculate the median unbiased estimates (as well as the 5 and 95 percentiles of the distribution) and report them in Tables 3 and 4 .

The results of the bias correction procedures they tell a similar story to what was found for the point estimates of the persistence parameter. The median unbiased estimates of Λ are lower for the LCP, CKM and CKM_IM specifications as compared to those found in the actual data. The versions of the model with home bias, non-traded goods, and CKM incorporating non-traded goods (CKM_NT) appear to generate sufficiently persistent real exchange rates.

3.4 Linearity vs. Nonlinearity

In addressing the second question of this paper, we now examine the extent to which the simulated real exchange rates exhibit nonlinear dynamics. Several recent papers have argued that a possible resolution to Rogoff's (1996) PPP puzzle is to model the real exchange rate as a nonlinear stationary process. Researchers have derived and estimated nonlinear time series models that feature international transactions costs, heterogeneity of agents' beliefs and misalignment in the foreign ex-

change market leading to a lack of coordination between market participants. The non-linearities have taken the form of Markov switching, threshold processes and variants of smooth transition (STAR) models. Although the nonlinear modeling strategies differ across papers, all such studies aim to show that small permanent deviations from PPP are possible due to frictions, whereas large deviations are quickly corrected. Consequently, real exchange rates exhibit mean reverting behavior only when there is a substantial deviation from the level implied by purchasing power parity.

As a first step to understanding the dynamics of the real exchange rates generated by DSGE models, we limit our attention to STAR type nonlinearity, given the popularity and success of that framework. The dynamics of the real exchange rate can be described as a STAR process as follows:

$$q_t = \sum_{j=1}^p \lambda_j q_{t-p} + \left(\sum_{j=1}^p \lambda_j^* q_{t-p} \right) F(q_{t-d}, \kappa) + u_t,$$

where the sequence of deviations from PPP, $\{q_t\}$, is a globally stationary ergodic process, and $u_t \sim iid(0, \sigma^2)$. $F(\cdot)$ represents a transition function from one regime to another and determines the degree of mean reversion. The parameters d and κ represent the delay and speed of adjustment parameters respectively.

A popular mechanism used to characterize this type of nonlinearity has led to the use of an ESTAR (exponential smooth transition) model, where the transition function above takes an exponential form:

$$F(\kappa, q_{t-d}) = 1 - \exp \left[-\kappa (q_{t-d} - \bar{q})^2 \right]$$

The exponential transition function above has the following property: $F : (0, \infty) \rightarrow [0, 1]$, $F(0) = 0$ and $\lim_{x \rightarrow \pm\infty} F(x) = 1$ and is symmetric around zero. An alternative specification of $F(\cdot)$ is the logistic transition function, which yields a logistic smooth transition autoregressive (LSTAR) model.

The transition function in this case can be written as:

$$F(\kappa, q_{t-d}) = \frac{1}{1 + \exp[-\kappa(q_{t-d} - \bar{q})]}$$

The ESTAR model has been shown to succinctly capture the dynamics of real exchange rates in the current floating period.

The popularity of the STAR framework arises from its ability to demonstrate how the real exchange rate may move smoothly from one type of regime to another, depending on how far the value of the real exchange rate is from a particular threshold. Small deviations from PPP are treated as being persistent by the STAR framework, whereas large deviations exhibit mean-reverting dynamics. In the ESTAR variant, positive and negative deviations from the threshold are treated symmetrically, whereas the LSTAR specification is indicative of asymmetric adjustment.

We proceed by testing for STAR-type nonlinearity vis-à-vis a linear specification described in (11). For each trial, we implement the method outlined in Teräsvirta (1994), which first involves determining the order of autoregression, p . We use the same values of p that were estimated for the linear case. The next steps test for linearity and select the value of the delay parameter, d , based on a Taylor expansion of the transition function around $\kappa = 0$. The final step requires choosing between an ESTAR and LSTAR specification, where we utilize the model selection criteria outlined by Teräsvirta (1994) that is most commonly used in the literature.¹⁴ We also employ an alternative method for choosing between the two specifications, that was proposed by Escribano and Jorda (2001), who develop a test that they claim has more power in selecting between the ESTAR and LSTAR alternatives. Our goal here is to determine the number of times that nonlinearity is indicated, and conditional on an indication of nonlinearity, the number of times that an ESTAR/LSTAR specification is chosen. Our results are reported in panel B of Table 4.

¹⁴We refer the reader to Teräsvirta (1994) for further details regarding the estimation methodology.

With the exception of the non-traded goods and incomplete markets versions of the model, nonlinearity is found more than two thirds of the time in the remaining cases, with home bias possessing the highest incidence of nonlinearity among the four basic specifications. The version of the model incorporating the trifecta of local currency pricing, home bias and non-traded goods (CKM_NT), which yielded the most volatile real exchange rate, also produces the highest number of non-linear trials (77 percent, just marginally above home bias). For the two versions of the model that yielded the most persistent real exchange rate - non-traded goods and incomplete markets, we see that nonlinearity is indicated approximately 55 and 60 percent of the times, respectively. Our results appear to weakly suggest that a trade-off might exist between persistence and nonlinearity.

To gain further intuition for this finding, we conjecture that the persistence and nonlinearity results are reconcilable if one considers the issue of small sample problems. If there are only a few observations of the real exchange rate in the outer regime where we detect mean reverting behavior, and the majority of the observations lie within the inner (persistent) regime, then a nonlinear data generating process may be estimated as a linear framework due to low power. This issue has been pointed out by Ahmad (2008) within the context of TAR models, which are a special case of the STAR framework. We believe that this is exactly what we are finding in the non-traded goods and incomplete markets cases, where the real exchange rates were found to be most persistent.

When looking at the type of nonlinearity indicated across the various versions of the model (conditional on STAR nonlinearity being detected), we find that both Teräsvirta's (1994) and Escribano and Jorda's (2001) methods indicate the predominance of asymmetric behavior in the form of an LSTAR model. However, in the versions of the model where the incidence of nonlinearity being detected is the greatest, namely the home bias and CKM_NT versions, we observe that a symmetric type of nonlinear dynamics in the form of an ESTAR model is favored.

Overall, when considering the results in Table 4 and using the criteria of volatility, persistence and nonlinearity, it appears that two versions of the model do reasonably well in matching the data: home bias and the CKM_NT specification that incorporates local currency pricing, home bias and non-traded goods. We investigate these versions further in section 4 to gain deeper understanding of the mechanisms behind our results.

3.5 Robustness Analysis: second-order approximation

Given the theoretical complexity and the presence of structural nonlinearity embedded in the DSGE models, analytical solutions are typically not feasible. Consequently, such models are usually solved by linearizing the equilibrium system of equations (to first order, in our case) around the model's steady state. A valid criticism of this solution methodology is that it may lead to a systematic bias and result in a lower incidence of nonlinearity being detected in the simulated real exchange rates series. To address this issue, we therefore perform a robustness check by solving the models outlined above using second order approximation, which may allow for further translation of any structural nonlinearity into STAR dynamics of real exchange rates. We report our findings in Table 5 and again focus on volatility, persistence and nonlinearity, in that order.

First, with regards to volatility, we observe an increase in volatility for the LCP, home bias and CKM_NT versions of the models vis-à-vis the first order solution. In the case of the CKM_NT, we note that the mean volatility across the thousand trials yields a value of 0.214, which is perfectly in line with what we observe in the actual data, and is in fact slightly higher than the average value of 0.17 across the countries that we examined. The astute reader at this point will note that the mean value in the CKM_NT case is higher than that of the value at the 95th percentile. For this particular version of the model, the volatilities appear to be highly skewed. The value of the volatility at the 99th percentile is 95.04, suggesting that the increase in the mean volatility is

resulting from a rightward skew of the distribution. The median volatility in the CKM_NT case is 0.103, and for the remaining versions of the model, the median volatilities are approximately equal to the mean once again, at least to three decimal places. Overall, the results here show that there are few gains in volatility to be obtained by using second order approximations.

Next, with regards to persistence, we find small increases for the LCP, home bias, CKM and CKM_IM versions of the model, where the increase in persistence is approximately 3% on average across these versions of the model. However, for the remaining cases, we see a decline in persistence on average of also approximately 3% across these models, with the greatest decline in persistence in the CKM_NT version of nearly 6%. Thus, we conclude that the greatest gains from second order approximation (in terms of absolute levels) for persistence are obtained in the home bias case.

Third, with regards to nonlinearity, we find, as expected, a greater incidence of nonlinearity across all the models, with the exception of LCP, where it remains exactly the same. The increased incidence of nonlinearity is not uniform across the models. With the exception of the pure incomplete markets specification, which we discuss below, the home bias and CKM_NT are once again the leading candidates exhibiting nonlinear behavior. When looking at the type of nonlinearity, we see that the conditional probability of symmetric (ESTAR) or asymmetric (LSTAR) behavior remains roughly the same for all versions of the model, although Teräsvirta's method now favors LSTAR behavior in the majority of cases (excepting home bias and CKM_NT as before).

Finally, we want to make a few observations for the case of the incomplete markets version of the model. We obtain highly persistent real exchange rates, second to only the non-traded goods specification. Moreover, we see a substantial increase in the incidence of nonlinearity, rising from 60% to 100%, and there is strong evidence to indicate that the nonlinearity is best described by the LSTAR process (the asymmetric type). However, our concern about the incomplete markets

version of the model relates to the volatility of the real exchange rate. The volatility appears to be negligible, and certainly not of an order comparable to the rest of the models that we investigate. We propose the following conjecture for these findings. Incomplete markets do not allow for perfect insurance against idiosyncratic shocks. So, in order to hedge against risk, exchange rates should be moving around a lot more than when financial markets are complete; yet, we see very little movement in this variable. Moreover, any resulting nonlinearities arising from the behavior of agents in the model are being captured as STAR type nonlinearity. We are forced to contend with the possibility that the nonlinearity we are observing in this scenario may be spurious, and simply an artifact of the second order approximation to the model's steady state, although we readily admit that we are not able to identify this with the methods we employ, provided that the conjecture is true. We simply note this as a possibility and therefore do not include this version among the leading candidates that we pursue in the next section.

4 Exploring the Sources of Non-Linearity

Home bias in consumption and the combination of pricing to market, home bias and non-traded goods (CKM_NT) emerged as the two specifications most likely to produce persistent and non-linear real exchange rate series, under both the first and second order approximations to the model's steady state. Although it is readily apparent that DSGE models at the micro level incorporate structural nonlinearities in the form of home bias, local currency pricing and non-traded goods (to mention a few), it is not quite so apparent how these features translate into macro-level nonlinearities of real exchange rates or other macroeconomic aggregates.¹⁵ Although this is a very difficult issue to address, we nevertheless proceed by investigating how the degree of home bias and the dynamics of non-traded goods sector affect the incidence of nonlinearity that we detect. In order to

¹⁵Moreover, nonlinearities imbedded in the primitive structure of the theoretical model may not be detected by the estimation techniques of the time series models.

do this, we study the impact of parameter calibration on the dynamic properties of the simulated real exchange rate series.

4.1 Home Bias and Nonlinearity

In our first exercise, we investigate the relationship between the degree of home bias and the dynamics properties of the real exchange rate. To that end, we create a grid of values of γ , the degree of consumption home bias, ranging from $\gamma = 0.50$ (no bias) to $\gamma = 0.99$ (almost complete home bias) in steps of 0.05. For each value of the parameter, we run 1000 simulations of the model, each generating 5000 observations of the real exchange rate. We record the incidence of nonlinearity detected, including the particular type of nonlinearity (symmetric ESTAR or asymmetric LSTAR), as well as persistence and volatility of the series. Our findings are illustrated in the top half of Figures 3 through 6.

Both volatility and persistence increase with the degree of home bias, since the real exchange rate must now work through fewer (tradable) goods prices to bring about market equilibrium in response to economic shocks. Real exchange rate volatility is a monotonically increasing function of γ (Figure 3), increasing rapidly for values of γ between 0.8 and 0.85. We report in Figure 4 that persistence on the other hand, is not a monotonic function of γ : it remains at approximately 0.75 for low values of home bias, and starts to increase rapidly when γ rises from 0.8 to 0.85.

Regarding the incidence of nonlinearity, Figure 5 indicates that it increases with the degree of home bias, although not at a constant rate. The maximum incidence occurs between $\gamma = 0.9$ and $\gamma = 0.95$. Moreover, both Teräsvirta's and Escribano and Jordá's methods show an increase in the probability of symmetric dynamic adjustment (Figure 6), since the conditional probability of an ESTAR model (given that nonlinearity is indicated) increases with γ . Thus, for lower values of this

parameter, $\gamma \in \{\gamma : 0.5 \leq \gamma \leq 0.8\}$, both model selection procedures favor LSTAR (asymmetric) dynamics of the real exchange rate; for $\gamma \in \{\gamma : 0.85 \leq \gamma \leq 0.99\}$, both model selection procedures favor an ESTAR process. For $0.8 \leq \gamma \leq 0.85$, the conditional probability of ESTAR or LSTAR is approximately half.

4.2 Non-traded Goods and Nonlinearity

We now explore how the addition of non-traded goods to the theoretical model impacts the behavior of real exchange rates. Several studies have found evidence of the Harrod (1933), Balassa (1964) - Samuelson (1964) effects in real exchange rate data (see, for example: Canzoneri, Cumby and Diba, 1999; Lothian and Taylor, 2008; and Ahmad and Craighead, 2011). Theoretically, in models with a non-traded sector, the real exchange rate can be written as a function of the relative movements in sectoral productivities across countries:

$$Q_t = f [(A_{T,t} - A_{N,t}) - (A_{T,t}^* - A_{N,t}^*)] \quad (13)$$

Countries that enjoy faster technological progress (which more often than not is concentrated in the tradables sector) consequently experience real exchange rate appreciation since rising costs of production factors (labor and capital) drive up prices in the non-tradable sector.

Motivated by this theoretical mechanism, we explore the impact of varying the relative volatility of technological shocks on the dynamic properties of the simulated real exchange rate series. Similarly to the methodology of the previous subsection, we create a grid of values of $\sigma_{A,N}$ (the variance of the shock to Home non-tradables) ranging from 50% to 200% of the tradable shock volatility, holding all other variances constant. More precisely, we vary the relative magnitude of the shock from 3.25×10^{-5} to 9.85×10^{-5} in steps of 0.15×10^{-5} . We also perform an analogous exercise by varying $\sigma_{A,T}$ while keeping $\sigma_{A,T}^*$, $\sigma_{A,N}$ and $\sigma_{A,N}^*$ at their benchmark values. For each value

of the parameter of interest, we generate 1000 simulations of the real exchange rate, each 5000 observations long, and analyze its volatility, persistence and the incidence of nonlinearity. The bottom panels of Figures 3 through 6 summarize the results for the former case (varying $\sigma_{A,N}$), since we find that there is not much variation in the incidence of nonlinearity from variation in productivity shocks of the tradable sector.

In figure 3, we find as predicted by (13), that the volatility of the real exchange rates is a monotonically increasing function of the non-tradable productivity variance. With regards to persistence reported in Figure 4, we find that it also increases (more so for $\sigma_{A,N} < \sigma_{A,T}$ than otherwise), matching the empirical estimates of this moment almost perfectly. However the increase is, in some sense, marginal, because we find that it remains in the neighborhood of 0.99, regardless of the relative size of the productivity shocks in the non-tradeable to tradeable sectors. We view this to indicate that the persistence being generated by the non-traded version of the model is inherent to the model's specification, rather than simply being an artifact of relative productivity shocks.

Figure 5 reports the results of the exercise on the incidence of nonlinearity. We note that the incidence of nonlinearity exhibits an inverted U-shape pattern centered in the neighborhood of $\sigma_{A,N} = \sigma_{A,T}$. In most of the cases where the size of the non-tradeable shocks is not equal to that of the tradable sector, we detect nonlinearity in only one third of cases (compared to 55% reported in Table 4). Moreover, the conditional probability of observing ESTAR dynamics in Figure 6 follows the same pattern as the incidence of nonlinearity itself, with its maximum (of 60 to 65%, depending on the method used) centered on the symmetric shock calibration. We conclude based on this evidence that incidence of nonlinearity, particular of the symmetric ESTAR variety, will be greatest when relative productivity shocks of non-tradeable to tradeable sectors are approximately equal in magnitude.

5 Conclusion

The goal of this paper has been to study the dynamics of real exchange rates generated by DSGE models that incorporate four often-used extensions: local currency pricing, home bias in consumption, non-traded goods and incomplete markets. More specifically, we examine this dynamics from two angles. First, we consider the extent to which real exchange rates in the new open economy DSGE models exhibit persistence and volatility relative to the data. Second, we check whether the dynamics of real exchange rates can be characterized as a nonlinear process, which we estimate using a smooth transition autoregressive approach.

Our key finding is that home bias and the presence of non-traded goods are the two features of DSGE models that are most likely to generate the persistence and nonlinearity of real exchange rates seen in the data. While other extensions like local currency pricing or incomplete markets do a poor job of replicating these properties, the combination of all these specifications perform reasonably well in matching the data from a business cycle perspective. Even though none of the four setups we examine do well in matching the volatilities of the raw data, the version of the model that combines local currency pricing, home bias and non-traded goods comes reasonably close.

Overall, we recognize that home bias, local currency pricing, incomplete markets and non-traded goods represent structural micro-level nonlinearities. However, these micro features need not, *a priori*, generate macro-level nonlinearities that can be detected by time series models. Our results suggest that we can identify some, but not others. Furthermore, the specifications of the model that do produce identifiable nonlinearities can be described by smooth transition autoregressive (STAR) processes. More specifically, we are able to characterize the nonlinear dynamics of real exchange rates as an ESTAR process for the vast majority of simulations that incorporate home bias, and, separately, the mix of local currency pricing, home bias and non-traded goods.

This translation of nonlinearities from structural primitives to the time series processes provides validation, in a sense, of the dynamic representations of time series models despite their lack of “micro-founded” theoretical underpinnings. Moreover, it also demonstrates that certain features of DSGE models allow for a reduced-form representation, in the form of time-series models, that captures the salient properties of the macroeconomic data.

Finally, although the choice of DSGE model specifications examined in this paper was motivated by a desire to bridge the two distinct strands of literature, as well as to better address the dynamics of real exchange rates, one could certainly suggest a variety of other features that could be considered. Extensions that include different exchange rate regimes, varying relative country size and dynamic portfolio choice, we leave to future work.

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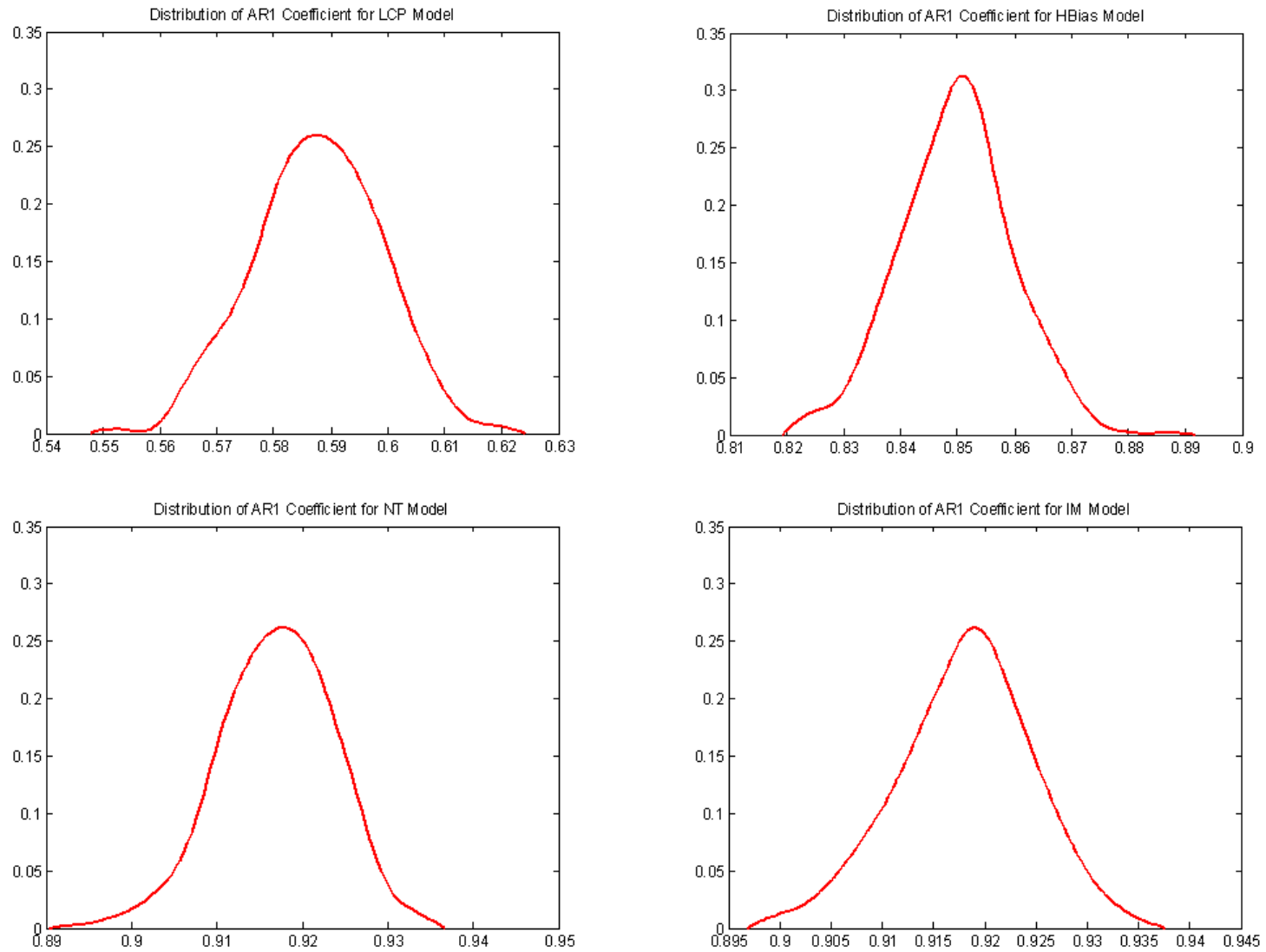


Figure 1. Distribution of AR(1) coefficients across various DSGE models.

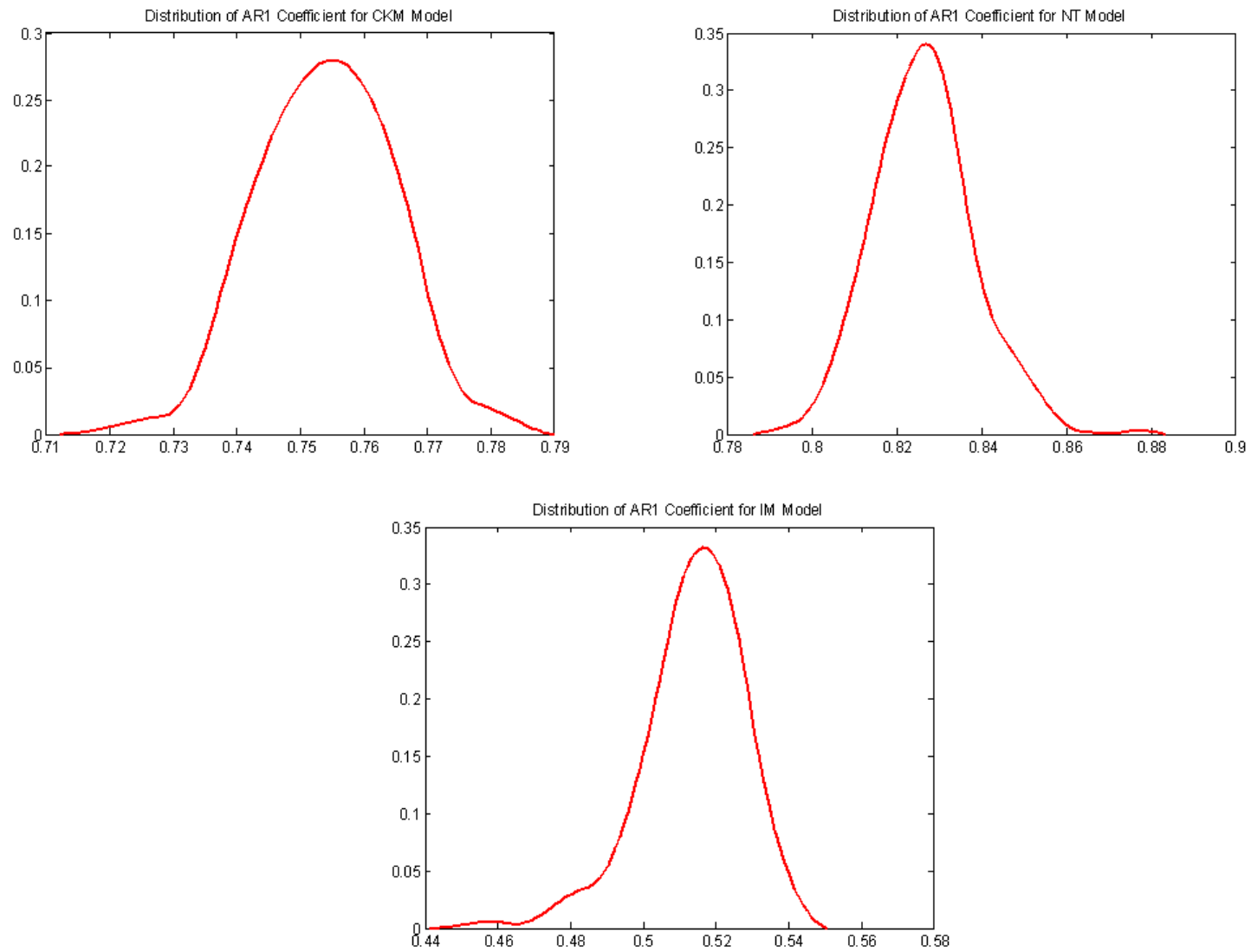


Figure 2. Distribution of AR(1) coefficients across CKM versions of DSGE model.

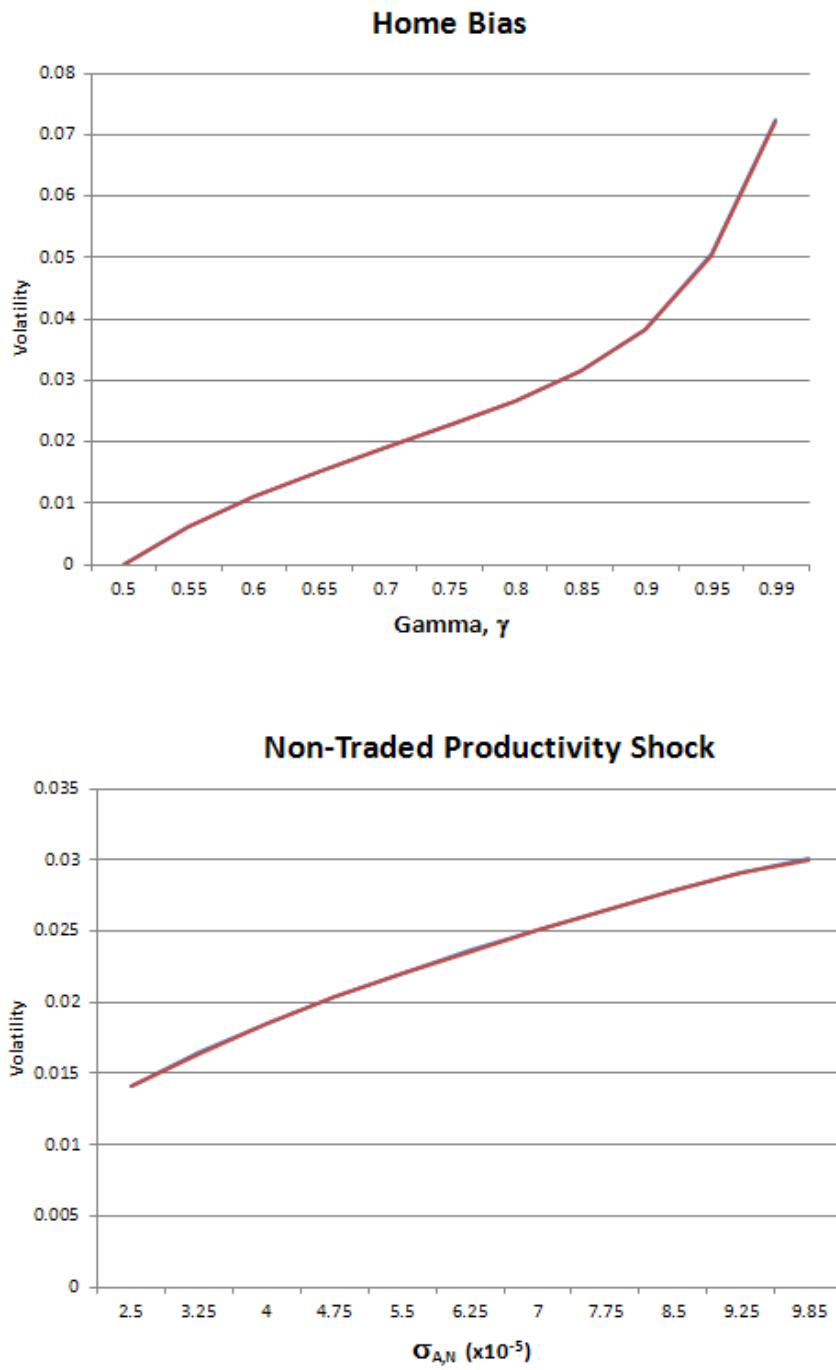


Figure 3. Volatility of simulated real exchange rates

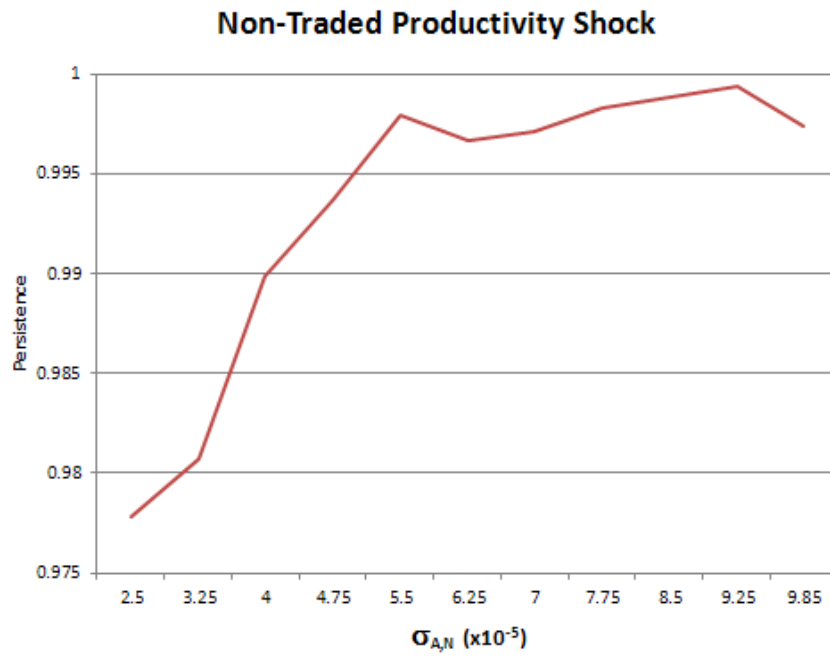
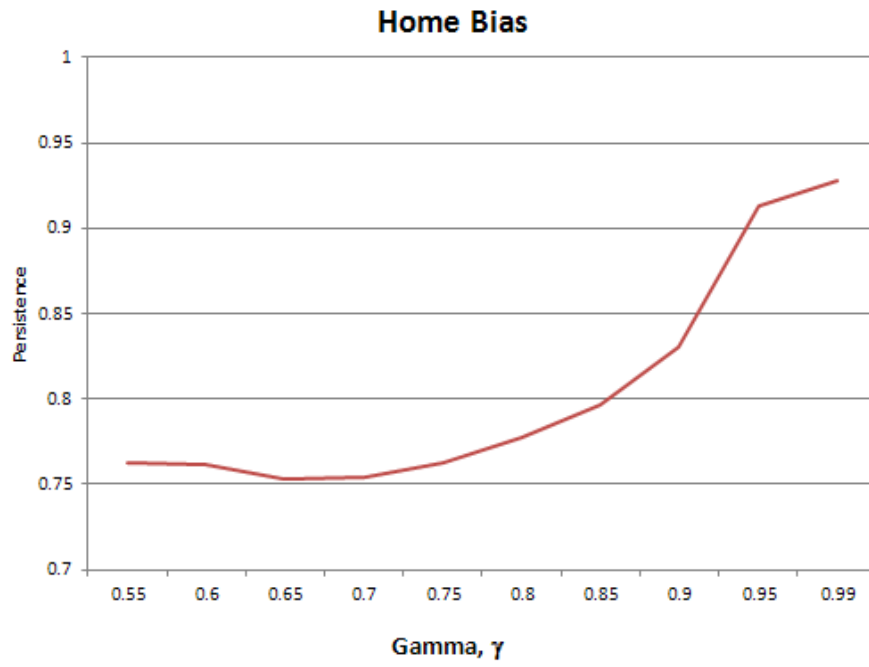


Figure 4. Median unbiased estimate of persistence of simulated real exchange rate.

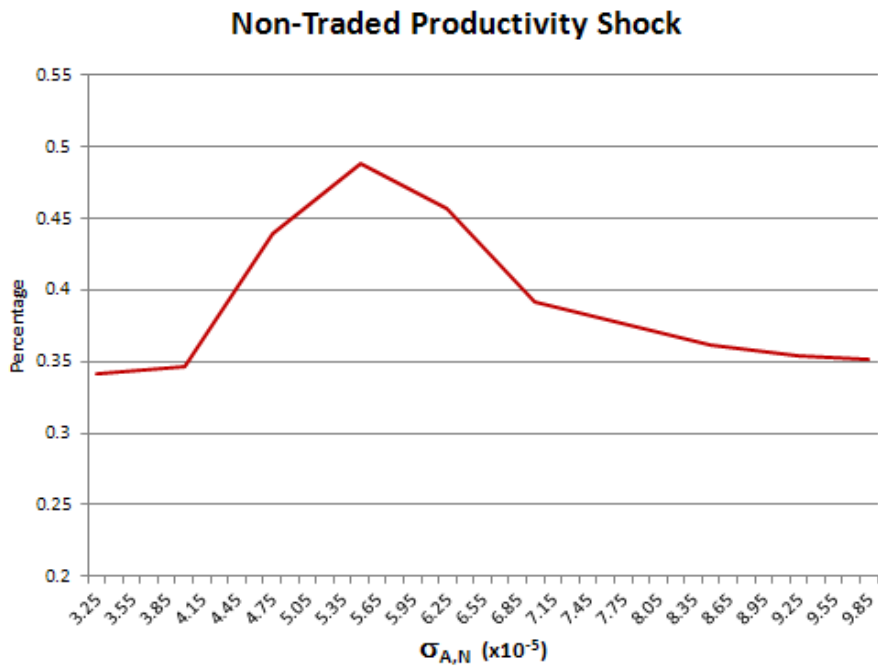
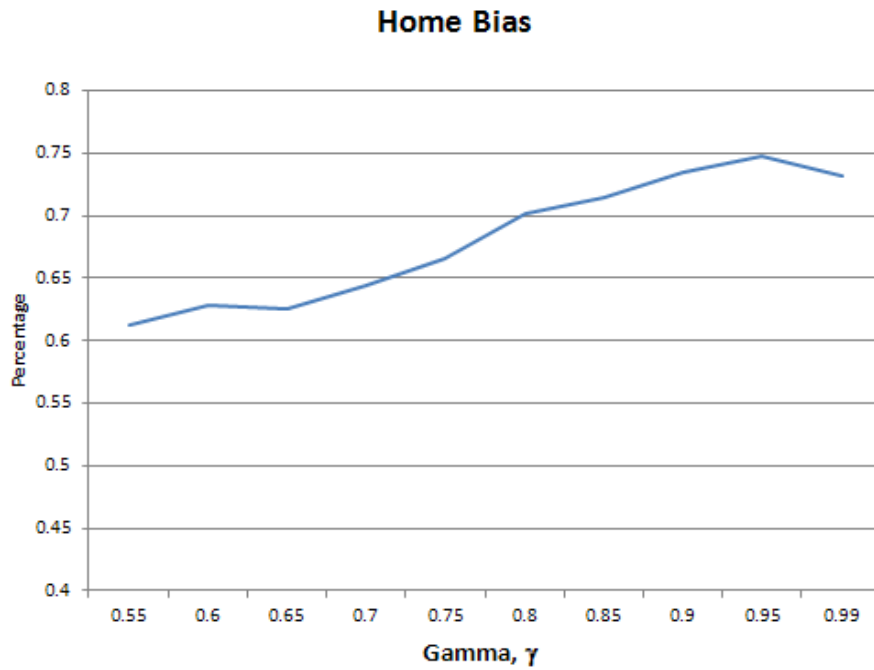


Figure 5. Percent of times STAR nonlinearity is detected

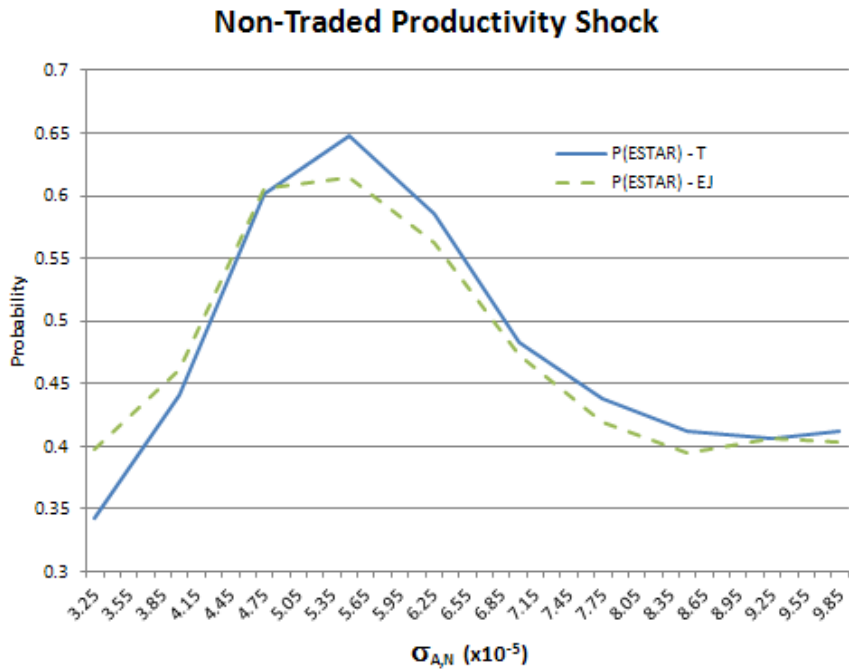
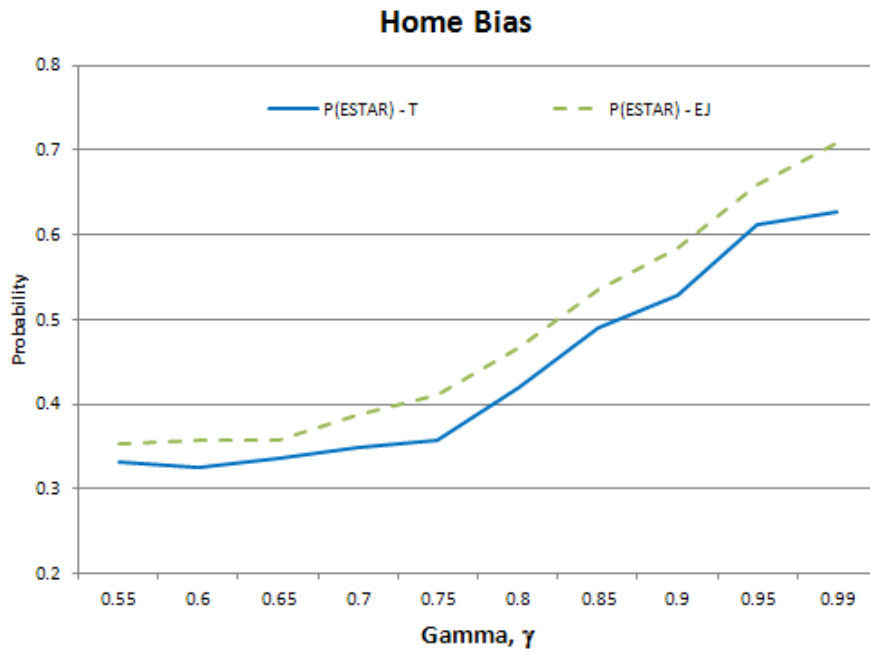


Figure 6. Conditional probability of ESTAR dynamics

Parameter	Description	Value
β	Household discount factor	0.99
Θ	Relative risk aversion	5.0
χ	Inverse of Frisch labor elasticity	3.0
θ	Elasticity of substitution between imports and exports	1.5
γ	Consumption home bias (0.5 in benchmark version)	0.94
μ	Weight on non-tradables in consumption	0.6
η	Elasticity of substitution between tradables and nontradables	0.74
A^T, A^N	Matrix of technology coefficients	$\begin{pmatrix} 0.95 & 0.00 \\ 0.00 & 0.95 \end{pmatrix}$
ν	Share of capital in production of tradables	0.33
ζ	Share of capital in production of nontradables	0.44
δ	Capital depreciation rate	0.021
σ_T, σ_N	Elasticity of substitution between goods varieties	10.0
ϕ	Elasticity of substitution between labor varieties	7.7
α, ω	Nominal stickiness	0.75
ρ_i	Interest rate inertia in the Taylor rule	0.79
ρ_π	Coefficient on inflation in the Taylor rule	2.15
ρ_y	Coefficient on output gap in the Taylor rule	0.23
p	International risk premium	0.01

Table 1: Benchmark parameter values

	Data	Bench	HBias	LCP	NT	IM	CKM	CKM_IM	CKM_NT
St. dev. GDP	1.70	1.68	1.69	1.70	1.69	1.70	1.67	1.71	1.71
St. dev. relative to GDP									
Consumption	0.86	0.88	0.87	0.88	0.76	0.91	0.88	0.88	0.88
Investment	2.59	1.70	1.83	1.87	1.88	1.74	2.15	2.39	1.98
Real exchange rate	3.14	0	1.75	2.33	1.11	0	3.62	1.90	4.16
Current account	0.13	-	-	-	-	0.19	-	0.12	-
Autocorrelations									
GDP	0.89	0.59	0.59	0.65	0.76	0.59	0.61	0.59	0.59
Consumption	0.88	0.59	0.59	0.58	0.77	0.59	0.59	0.59	0.58
Investment	0.91	0.59	0.59	0.58	0.77	0.59	0.58	0.58	0.58
Real exchange rate	0.79	0.59	0.61	0.47	0.92	0	0.56	0.43	0.59
Cross-correlations									
U.S.-UK GDP	0.53	0.53	0.52	0.53	0.51	0.52	0.52	0.52	0.53
U.S.-UK Consumption	0.43	1.00	0.92	0.86	0.96	0.96	0.66	0.49	0.55
Real exchange rate and GDP	-0.16	0	0.48	0.25	0.30	0	0.46	0.43	0.38

Table 2: Business cycle properties of the models. The data refer to the U.S. and UK variables over the 1975Q2-2010Q3 period. All data and simulated series are logged and HP-filtered. Since the path of current account does not affect any allocations when asset markets are complete, we only report its volatility for the two incomplete markets specifications (IM and CKM_IM).

	Canada	France	Germany	Italy	Japan	UK
Autocorrelation	0.968	0.929	0.928	0.936	0.958	0.927
ADF Test	-1.603	-2.436	-2.446	-1.964	-2.721*	-2.647*
PP Test	-1.606	-2.289	-2.342	-1.831	-2.535	-2.728*
λ_1	1.168	0.942	0.9403	0.957	1.21	1.097
Λ	0.965	0.942	0.9403	0.957	0.958	0.917
p	2	1	1	1	2	2
σ_{RER}	0.122	0.163	0.175	0.167	0.254	0.149
LJB Test	3.82	1.94	1.22	1.79	4.63	4.37
Λ_{MUE}	0.998	0.975	0.982	0.9752	0.9829	0.9474
[5% 95%]	[0.954 1.028]	[0.911 1.021]	[0.91 1.022]	[0.932 0.996]	[0.933 1.018]	[0.870 1.011]

Table 3: Time series properties of the real exchange rate for different countries between 1970Q1-2010Q1. Notes: The ADF and PP tests report the value of the augmented Dickey-Fuller and Phillips Perron test statistics, respectively; (*) represents significance at the 10 percent level. MUE represents the median unbiased estimate of Λ . Figures in square brackets represents the 5 and 95 percentiles for the median unbiased estimate of the persistence parameter, computed using Stock's (1991) method.

	LCP	HBias	Non-Traded	Incomplete Markets	CKM	CKM+Non-Traded (CKM_NT)	CKM+Incomplete Markets (CKM_IM)
<i>Panel A: Persistence and Volatility</i>							
p	1	2	2	1	1	2	1
[5% 95%]	[1 1]	[1 5]	[1 4]	[1 4]	[1 2]	[1 5]	[1 3]
Mean Volatility	0.045	0.047	0.023	3.44×10^{-16}	0.080	0.107	0.036
[5% 95%]	[0.044 0.046]	[0.045 0.049]	[0.022 0.025]	[3.24 3.68] $\times 10^{-16}$	[0.077 0.083]	[0.102 0.112]	[0.035 0.037]
Λ	0.595	0.846	0.918	0.930	0.751	0.852	0.527
Λ_{MUE}	0.614	0.875	0.960	0.969	0.782	0.906	0.544
[5% 95%]	[0.50 0.76]	[0.80 0.99]	[0.87 1.02]	[0.89 1.02]	[0.70 0.90]	[0.77 1.00]	[0.40 0.70]
<i>Panel B: Nonlinearity</i>							
Linearity	0.33	0.24	0.45	0.40	0.29	0.23	0.29
STAR	0.68	0.76	0.55	0.60	0.71	0.77	0.71
Teräsvirta's Method:							
$P(LSTAR STAR)$	0.60	0.35	0.52	0.48	0.52	0.42	0.59
$P(ESTAR STAR)$	0.40	0.65	0.48	0.52	0.48	0.58	0.41
EJ's Method:							
$P(LSTAR STAR)$	0.58	0.35	0.57	0.38	0.51	0.39	0.58
$P(ESTAR STAR)$	0.42	0.65	0.43	0.62	0.49	0.61	0.42

Table 4: Time series properties of the simulated real exchange rate from the different models.

Notes: Simulations are based on 5500 observations of simulated data for 1000 trials of the simulation, where we burn the first 500 observation to avoid the initial value bias. MUE represents the median unbiased estimate of Λ , computed using the method by Andrews (1993) and Andrews and Chen (1994). Figures in square brackets represent the 5 and 95 percentiles for the distribution of the autoregressive order and for the confidence interval for the median unbiased estimate of the persistence parameter, respectively, where the latter is computed using Stock's (1991) method.

	LCP	HBias	Non-Traded	Incomplete Markets	CKM	CKM+Non-Traded (CKM_NT)	CKM+Incomplete Markets (CKM_IM)
<i>Panel A: Persistence and Volatility</i>							
p	1	2	2	3	1	2	1
[5% 95%]	[1 1]	[1 4]	[1 4]	[2 5]	[1 2]	[1 5]	[1 3]
Mean Volatility	0.046	0.050	0.023	2.68×10^{-16}	0.082	0.214	0.036
[5% 95%]	[0.045 0.047]	[0.047 0.054]	[0.022 0.025]	$[2.471 \ 2.897] \times 10^{-16}$	[0.079 0.086]	[0.098 0.110]	[0.035 0.037]
Λ	0.609	0.868	0.914	0.901	0.764	0.823	0.541
Λ_{MUE}	0.634	0.902	0.951	0.939	0.794	0.852	0.564
[5% 95%]	[0.50 0.78]	[0.81 1.00]	[0.87 1.01]	[0.85 1.01]	[0.68 0.91]	[0.75 0.97]	[0.42 0.70]
<i>Panel B: Nonlinearity</i>							
Linearity	0.33	0.16	0.43	0.00	0.23	0.21	0.28
STAR	0.68	0.84	0.57	1.00	0.77	0.79	0.72
Teräsvirta's Method:							
$P(LSTAR STAR)$	0.65	0.43	0.50	0.80	0.53	0.41	0.60
$P(ESTAR STAR)$	0.35	0.57	0.50	0.20	0.47	0.59	0.40
EJ's Method:							
$P(LSTAR STAR)$	0.64	0.29	0.42	0.94	0.49	0.37	0.61
$P(ESTAR STAR)$	0.36	0.71	0.58	0.06	0.51	0.63	0.39

Table 5: Time series properties of the simulated real exchange rate from the different models for second order approximations.

Notes: Simulations are based on 5500 observations of simulated data for 1000 trials of the simulation, where we burn the first 500 observation. MUE represents the median unbiased estimate of Λ , computed using the method by Andrews (1993) and Andrews and Chen (1994). Figures in square brackets represent the 5 and 95 percentiles for the distribution of the autoregressive order and for the confidence interval for the median unbiased estimate of the persistence parameter, respectively, where the latter is computed using Stock's (1991) method.

A First Order Conditions

Below we present the first order conditions for Home agents; analogous equations hold for firms and households in the Foreign economy.

A.1 Households

Benchmark model:

$$\begin{aligned}
\frac{1}{1+i_t} &= \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\Theta} \frac{1}{\Pi_{t+1}} \right] \\
C_t^{-\Theta} &= \xi_t \left[1 - \psi \left(\frac{I_t}{K_{t-1}} - \delta \right) \right] \\
\xi_t &= \beta E_t \left\{ C_t^{-\Theta} P_t^{-1} R_{t+1} + \xi_{t+1} \left[(1-\delta) - \frac{1}{2} \psi \left(\frac{I_{t+1}}{K_t} - \delta \right)^2 + \psi \left(\frac{I_{t+1}}{K_t} - \delta \right) \frac{I_{t+1}}{K_t} \right] \right\} \\
\tilde{W}_t^{\phi\chi+1} &= \frac{\phi}{\phi-1} \frac{E_t \sum_{j=t}^{\infty} (\omega\beta)^{j-t} \left\{ W_j^{\phi(\chi+1)} N_j^{1+\chi} \right\}}{E_t \sum_{j=t}^{\infty} (\omega\beta)^{j-t} \left\{ [C_j^{\Theta} P_j]^{-1} W_j^{\phi} N_j \right\}}
\end{aligned}$$

Incomplete markets:

$$1 = \beta E_t \left\{ [1 + i_t^* - p(e^{a_t - \bar{a}} - 1)] \left(\frac{C_{t+1}}{C_t} \right)^{-\Theta} \frac{\Delta S_{t+1}}{\Pi_{t+1}} \right\}$$

A.2 Firms

Benchmark model:

$$\begin{aligned}\tilde{P}_{H,t} &= \frac{\sigma_T}{\sigma_T - 1} \frac{E_t \sum_{j=t}^{\infty} [C_j^{\ominus} P_j]^{-1} (\alpha\beta)^{j-t} P_{H,j}^{\sigma_T} [Y_{H,j}^d + Y_{H,j}^{*d}] MC_j}{E_t \sum_{j=t}^{\infty} [C_j^{\ominus} P_j]^{-1} (\alpha\beta)^{j-t} P_{H,j}^{\sigma_T} [Y_{H,j}^d + Y_{H,j}^{*d}]} \\ MC_t &= \frac{R_t^{\nu} W_t^{1-\nu}}{A_{T,t} \nu (1-\nu)^{1-\nu}} \\ A_{T,t} K_{T,t-1}^{\nu} N_{T,t}^{1-\nu} &= [Y_{H,t} + Y_{H,t}^*] DP_{H,t} \\ DP_{H,t} &= (1-\alpha) \left[\frac{\tilde{P}_{H,t}}{P_{H,t}} \right]^{-\sigma_T} + \alpha \left[\frac{P_{H,t}}{P_{H,t-1}} \right]^{\sigma_T} DP_{H,t-1}\end{aligned}$$

Local currency pricing:

$$\begin{aligned}\tilde{P}_{H,t} &= \frac{\sigma_T}{\sigma_T - 1} \frac{E_t \sum_{j=t}^{\infty} [C_j^{\ominus} P_j]^{-1} (\alpha\beta)^{j-t} P_{H,j}^{\sigma_T} Y_{H,j} MC_j}{E_t \sum_{j=t}^{\infty} [C_j^{\ominus} P_j]^{-1} (\alpha\beta)^{j-t} P_{H,j}^{\sigma_T} Y_{H,j}} \\ \tilde{P}_{H,t}^* &= \frac{\sigma_T}{\sigma_T - 1} \frac{E_t \sum_{j=t}^{\infty} [C_j^{\ominus} P_j]^{-1} (\alpha\beta)^{j-t} P_{H,j}^{*\sigma_T} Y_{H,j}^* MC_j}{E_t \sum_{j=t}^{\infty} [C_j^{\ominus} P_j]^{-1} (\alpha\beta)^{j-t} P_{H,j}^{*\sigma_T} Y_{H,j}^* S_j} \\ A_{T,t} K_{T,t-1}^{\nu} N_{T,t}^{1-\nu} &= Y_{H,t} DP_{H,t} + Y_{H,t}^* DP_{H,t}^*\end{aligned}$$

Non-traded goods:

$$\begin{aligned}\tilde{P}_{N,t} &= \frac{\sigma_N}{\sigma_N - 1} \frac{E_t \sum_{j=t}^{\infty} [C_j^{\ominus} P_j]^{-1} (\alpha\beta)^{j-t} P_{N,j}^{\sigma_N} Y_{N,j} MC_{N,j}}{E_t \sum_{j=t}^{\infty} [C_j^{\ominus} P_j]^{-1} (\alpha\beta)^{j-t} P_{N,j}^{\sigma_N} Y_{N,j}} \\ MC_{N,t} &= \frac{R_t^{\zeta} W_t^{1-\zeta}}{A_{N,t}^{\zeta} \zeta^{\zeta} (1-\zeta)^{1-\zeta}} \\ Y_{N,t} DP_{N,t} &= A_{N,t} K_{N,t-1}^{\zeta} N_{N,t}^{1-\zeta} \\ DP_{N,t} &= (1-\alpha) \left[\frac{\tilde{P}_{N,t}}{P_{N,t}} \right]^{-\sigma_N} + \alpha \left[\frac{P_{N,t}}{P_{N,t-1}} \right]^{\sigma_N} DP_{N,t-1}\end{aligned}$$