Geometry Concepts

ANGLES

Angles can be classified as:

- ***acute*** (less than 90^o)
- ***right*** (equal to 90^o, a square corner)
- ***obtuse*** (greater than 90^o but less than 180^o)
- ***straight*** (equal to 180^o, a straight line).

Angles are measured with a ***protractor***.

Complementary angles are two angles whose measures add up to 90^o.
Supplementary angles are two angles whose measures add up to 180^o
Adjacent angles lie next to each other, sharing a side and a vertex.
Linear pairs are two angles whose measures add up to 180^o…they lie along a straight line.
Vertical angles are formed when two lines intersect…they are the angles that are opposite each other. **Vertical angles are congruent**.

LINES

Lines are measured with a ruler. Common ***metric measures*** are:

- ***Meter:*** a little more than a yard
- ***Centimeter:*** a little less than half an inch. There are 100 centimeters in a meter.
- ***Millimeter:*** about the thickness of a pencil lead. There are 1000 millimeters in a meter. There are 10 millimeters in a centimeter.

Perpendicular lines intersect to form a 90^o angle. Symbol is $\perp$
Parallel lines never intersect. They are the same distance apart everywhere. Symbol is $\| \|$
A ***bisector*** is a line that cuts an angle or line segment in half.

SYMMETRY

A figure has ***symmetry*** if a line can be drawn that divides the figure into mirror images: one side of the line is the exact mirror image of the other side of the line

CONGRUENT

Congruent means "same size, same shape." The word "congruent" is used when referring to shapes: we say two triangles are *congruent* if they are the *same size and same shape*. The symbol for congruent is $\cong$.

(The word "equal" is used when referring to a distance or length…we say that two *measures* are *equal*: 4 meters = 400 centimeters)

CONGRUENT TRIANGLES

Two triangles can be proven to be congruent using these postulates:

- SSS: Side-Side-Side
- SAS: Side-Angle-Side (the angle must be between the two sides)
- ASA: Angle-Side-Angle (the side must be between the two angles)
- AAS: Angle-Angle-Side (two sides and the angle NOT between them)
- HL: Hypotenuse-Leg (in a right triangle, the hypotenuse and one of the legs)

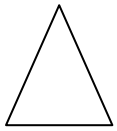
TRIANGLES:

Triangles are named by:

- Sides **Scalene**: No sides equal
  Isosceles: Two sides equal
  Equilateral: All sides equal

- Angels **Acute:** All angles less than 90^o
  Right: One angle = 90^o
  Obtuse: One angle greater than 90^o
  Equiangular: All angles are equal (all angles are 60^o)
  (Equiangular triangles are also equilateral)

Isosceles Triangles: In an isosceles triangle, the angles opposite the equal sides are congruent, and the sides opposite the congruent angles are equal.



Sides: In triangles, the sum of the shorter two sides will always be greater than the third side.

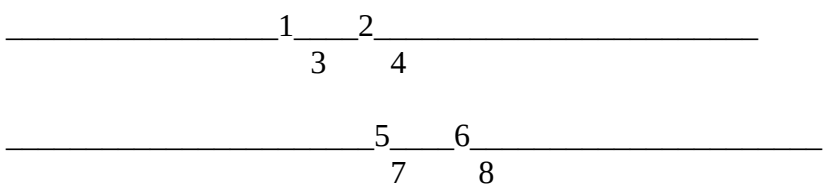
Side-Angle Relationship: In triangles, the shortest side is always opposite the smallest angle; the longest side is always opposite the largest angle. Conversely, the smallest angle is always opposite the shortest side, and the largest angle is always opposite the longest side.

Similar Triangles are the same shape, but not the same size. Similar triangles have the same angle measures, and their sides are proportional. Symbol is ~

PARALLEL LINES:

When two parallel lines are cut by a transversal, the angles are designated:

- Exterior: angles 1, 2, 7, 8
- Interior: angles 3, 4, 5, 6
- Alternate interior: 3 & 6; 4 & 5
- Alternate exterior: 1 & 8; 2 & 7
- Corresponding: 1 & 5; 2 & 6; 3 & 7; 4 & 8



Parallel Line Postulates: When two parallel lines are cut by a transversal:

- Alternate exterior angles are congruent
- Alternate interior angles are congruent
- Corresponding angles are congruent
- Interior angles on the same side of the transversal add to 180^o

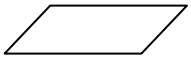
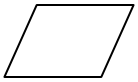
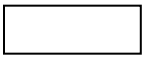
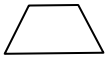
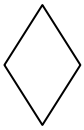
POLYGONS

Polygons are two-dimensional closed figures who sides are line segments. Polygons are named:

- 3-sided: triangle
- 4-sided: quadrilateral
- 5-sided: pentagon
- 6-sided: hexagon
- 7-sided: heptagon
- 8-sided: octagon
- 9-sided: nonagon
- 10-sided: decagon
- 12-sided: dodecagon
- *n*-sided: *n*-gon

Regular Polygons have all sides congruent, all angles congruent

Quadrilaterals are 4-sided polygons.

- ***Parallelograms*** have opposite sides that are parallel and congruent 
- ***Rhombus*** have opposite sides parallel, all sides congruent 
- ***Rectangles*** are parallelograms with four 90^0 angles 
- ***Squares*** are rectangles with four congruent sides 
- ***Trapezoids*** have one pair of parallel sides 
- ***Kites*** have two pair of adjacent congruent sides 

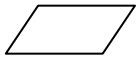
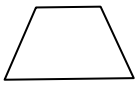
Angle measures of a polygon: if the number of sides of the polygon is called "n", then:

- The sum of the interior angle measures is found by $(n-2)180^0$
  - Triangles contain 180^0
  - Quadrilaterals contain 360^0
- To find the measure if EACH interior angle in a regular polygon: $\frac{(n-2)180}{n}$
- The sum of the exterior angle measures is always 360^0

Perimeter is the distance around the outside edge of a polygon. To find the perimeter, add up the lengths of all sides.

Area is a measure of how many "squares" cover a surface. The label for area is always "units squared"

Some common area and perimeter formulas: (*b = base, h = height, l = length, w = width, s = side)*

-  Rectangle: $A = bh$ *or* $A = lw$ $P = 2l + 2w$
-  Square: $A = s^2$ $P = 4s$
-  Parallelogram: $A = bh$ where $h \perp b$
-  Triangle: $A = \frac{1}{2}bh$ *or* $\frac{bh}{2}$ where $h \perp b$
-  Trapezoid: $A = \left(\frac{b_1 + b_2}{2}\right)h$ where $h \perp b$

CIRLCES (r = radius, from center to outside edge; d = diameter, edge to edge going through center)

-  Circle: $A = \pi r^2$ where $\pi \approx 3.14$ $C = \pi d = 2\pi r$

VOLUME

Volume is the measure of the space inside a 3-dimensional figure: how many "cubes" are inside a figure. The label for volume is always "units cubed".

The general volume formula is $\boldsymbol{V = Bh}$, where *B* is the area of the base of the figure. More specifically, the volumes of some common shapes are:

 Rectangular prism (box): $V = lwh$

 Cube: $V = s^3$

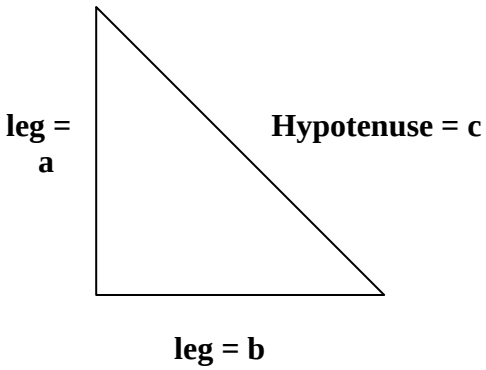
 Cylinder: $V = \pi r^2 h$

 Sphere: $V = \frac{4}{3}\pi r^3$ Surface Area: $SA = 4\pi r^2$

Right Triangle Geometry

The study of **Right Triangles** is an intricate part of the study of mathematics.
Right Triangles are often found in architecture and every day shapes.

Every right triangle has one 90^o angle. The side opposite the right angle is called the **hypotenuse…**the hypotenuse is always the longest side of a right triangle. The other two sides of the right triangle are called the **legs.**



Pythagorean's Theorem gives the relationship between sides of a right triangle:

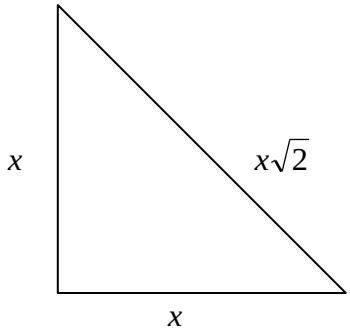
$$a^2 + b^2 = c^2$$

Use Pythagorean's Theorem to find a missing side of a right triangle if two of the sides are given:

- *****If the two legs are given, add their squares, then take the $\sqrt{\ }$ of their sum to get the hypotenuse.
- *****If the hypotenuse and one of the legs are given, subtract the squares, then take the $\sqrt{\ }$ of their difference to get the remaining leg.

Special Right Triangle: the Isosceles Right Triangle 45^o - 45^o - 90^o

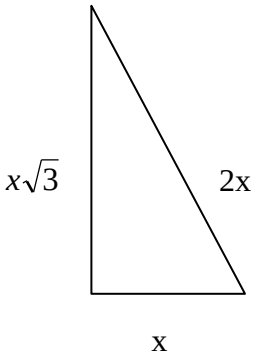
Two of the sides are the same length (congruent), therefore the angles opposite the congruent sides are also congruent, making them each 45^o.



If the congruent sides are given, the hypotenuse is found by multiplying one of the congruent sides times $\sqrt{2}$.

If the hypotenuse is given, then the congruent sides are found by dividing the hypotenuse by $\sqrt{2}$.

Special Right Triangle: 30^o - 60^o - 90^o



The hypotenuse is always twice as long as the short side. The medium-length side is found by taking the short side times $\sqrt{3}$.

If the medium-length side is given, divide its length by $\sqrt{3}$ to obtain the short side, then double the short side to obtain the hypotenuse.